# INTERFERENCE RESISTANT BLIND ACQUISITION AND CHANNEL ESTIMATION FOR CDMA COMMUNICATION SYSTEMS

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### ABSTRACT

The time scale over which a channel is observed determines whether a deterministic or statistical characterization is appropriate. We show that the channel estimation algorithm of Tsatsanis and Xu and the acquisition algorithm of Sayeed and Aazhang share a common MMSE formulation. They differ only in the assumption of the channel characterization. From this observation we develop a new blind MMSE interference resistant algorithm for both timing acquisition and channel estimation that is applicable to both deterministic and statistical channel models. Simulations illustrate the effectiveness of the new method.

## 1. INTRODUCTION

Accurate timing and channel estimation are critical to demodulation in wireless communication systems. In code-division multiple access (CDMA) systems this amounts to the estimation of a global delay offset and corresponding channel coefficients for RAKE combining. The presence of multiaccess interference (MAI) makes channel estimation particularly challenging. Conventional CDMA receivers treat MAI as background noise and rely on correlator-based timing acquisition. However, the performance of conventional receivers significantly degrades in the presence of strong MAI that may be encountered in highly mobile scenarios. A variety of MAI-resistant acquisition and channel estimation techniques have been proposed recently that exploit the structure of interference, including nonlinear MUSIC-based methods and linear MMSE filtering approaches [1, 2, 3, 4]. This paper focusses on linear filtering methods due to their simplicity and potentially better performance under low SNR scenarios [5].

Detailed characterization of the channel depends on the relative time scales over which the modulation and channel used for communication varies. Very slowly varying channels may be well modeled by FIR filters with fixed coefficients and coherent modulation/reception is appropriate. However, if the channel varies sufficiently rapidly, then noncoherent methods must be used and statistical characterization of the channel is more appropriate. The acquisition and channel estimation method presented here applies to both of these situations.

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Of particular interest to this paper are the methods of Tsatsanis and Xu [1] and Sayeed and Aazhang [2]. In the channel estimation method of [1] the channel is modeled by an FIR filter and the channel coefficients chosen to maximize the minimum output energy of a minimum variance receiver. This philosophy can also be used to determine the time of the first multipath arrival. The focus of [2] is timing acquisition and the channel is modeled using second-order statistics. The timing is acquired by minimizing the output power of a constrained quadratic processor. This method may also be used to estimate channel statistics.

In Section III of this paper we relate the methods of [1] and [2], and show that they differ only in the assumptions of deterministic and statistical channel models. Analysis indicates that both fail when their respective channel assumptions are violated. We then present a minimum mean squared error acquisition and channel estimation procedure that generalizes both [1] and [2], and is applicable to either deterministic or statistical channel models. Simulations illustrating the advantages of the new approach are given in Section IV.

## 2. SIGNAL REPRESENTATION

In general we may represent the received signal as a linear combination of fixed basis functions corresponding to space (angle of arrival), time (multipath delay), and frequency (Doppler shifts) [6]. For ease of exposition, we limit this paper to the multipath dimension. It is straightforward to extend the results given here to the full space-time-frequency problem. The received signal $x(t)$ due to a single bit $b$ from the *desired* user may be expressed as

$$x(t) = b \sum_{m=0}^{M-1} a_m s(t - (L+m)T_c) \tag{1}$$

where $s(t)$ is the spread spectrum signaling waveform with spreading gain $N$, $T_c$ is the chip duration, $L$ is the timing delay in chips, and $M-1$ denotes the multipath spread in chips. This implies the channel is represented by an $M$ tap FIR filter with coefficients $a_m$ and tap spacing $T_c$.

The received signal is match-filtered with the desired user's signalling waveform and sampled to obtain the canonical multipath coefficients (CMCs)

$$z_k = \int x(\tau) s^*(\tau - kT_c) d\tau \tag{2}$$

Substituting (1) and assuming a unit amplitude rectangular chip waveform, we obtain the contribution due to the desired user as

$$z_k = bT_c \sum_{m=0}^{M-1} a_m \rho(k - (L+m)) \tag{3}$$

where $\rho(l)$ is the cross-correlation function of the code at integer multiples of $T_c$. For an ideal code we have $\rho(0) = N$ and $\rho(l) = 0, l \neq 0$. In this case (3) becomes

$$z_k = \begin{cases} bNT_c a_k & L \leq k \leq L+M-1 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

That is, the channel effects are confined to the $L^{th}$ through $(L+M-1)^{th}$ CMCs. In practice the desired user's code will have small but nonzero correlation and the channel will have a small contribution to all CMCs. This contribution can be removed by projection to the space orthogonal to the desired user's code correlation. Noise and interfering users will contribute to all CMCs.

Let the length $N$ vector $\mathbf{z}(n)$ denote the CMCs for the $n^{th}$ symbol. The signal of interest is contained primarily in the $L^{th}$ through $(L+M-1)^{th}$ elements of $\mathbf{z}(n)$, termed the "active" CMCs. If the symbol $\mathbf{z}_L(n)$ denotes $\mathbf{z}(n)$ after a shift of $L$ and the length $M$ vector $\mathbf{h}(n)$ contains the corresponding CMCs of the desired user for symbol $n$, then the desired user contributes $b\mathbf{h}(n)$ to the first $M$ elements of $\mathbf{z}_L(n)$, which now are the active CMCs. The timing acquisition problem is to estimate the delay $L$ while the channel estimation problem consists of estimating either $\mathbf{h}(n)$ or the statistics of $\mathbf{h}(n)$ and possibly the spread $M$.

## 3. CONSTRAINED MMSE TIMING ACQUISITION AND CHANNEL ESTIMATION

The first two acquisition procedures considered are based on minimizing the output power of a linear receiver. That is, they involve minimizing $E\left\{|\mathbf{w}^H \mathbf{z}_L(n)|^2\right\}$ subject to a constraint on the weight vector $\mathbf{w}$.

### 3.1. Linearly Constrained Approach

If the channel is assumed to be very slowly varying, such that $\mathbf{h}(n) \approx \mathbf{h}$, then the linearly constrained minimum variance (LCMV) approach suggested in [1] is applicable. Let $\mathbf{h}_L^H = [\mathbf{h}^H 0 \ldots 0]$ be a length $N$ vector constructed by placing the unknown CMCs for the desired user in the first $M$ elements. The LCMV receiver chooses $\mathbf{w}$ to satisfy

$$\min_{\mathbf{w}} E\left\{\left|\mathbf{w}^H \mathbf{z}_L(n)\right|^2\right\} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{h}_L = 1. \tag{5}$$

If we let $\mathbf{R}_L = E\left\{\mathbf{z}_L(n)\mathbf{z}_L^H(n)\right\}$, then the weight vector is given by

$$\mathbf{w} = \frac{\mathbf{R}_L^{-1}\mathbf{h}_L}{\mathbf{h}_L^H \mathbf{R}_L^{-1}\mathbf{h}_L} \tag{6}$$

and the output power is

$$P(L, M, \mathbf{h}) = \left(\mathbf{h}_L^H \mathbf{R}_L^{-1}\mathbf{h}_L\right)^{-1} \tag{7}$$

where we have explicitly denoted the dependence of the output power on the unknown values of $L$ and $\mathbf{h}$. The minimization step suppresses interference, so we estimate $L$, $M$, and $\mathbf{h}$ as the values that maximize $P(L, M, \mathbf{h})$.

It is insightful to write (7) in a different form. First partition $\mathbf{z}_L(n)$ as

$$\mathbf{z}_L(n) = \begin{bmatrix} \mathbf{z}_A(n) \\ \mathbf{z}_I(n) \end{bmatrix} \tag{8}$$

where $\mathbf{z}_A(n)$ and $\mathbf{z}_I(n)$ represent the first $M$ and last $N-M$ elements of $\mathbf{z}_L(n)$. If $L$ and $M$ are correct, then $\mathbf{z}_A(n)$ represents the active CMCs containing the desired user's signal and $\mathbf{z}_I(n)$ represents inactive CMCs containing interfering users' signals and noise. The corresponding partition of $\mathbf{R}_L$ is

$$\mathbf{R}_L = \begin{bmatrix} \mathbf{R}_A & \mathbf{R}_{AI} \\ \mathbf{R}_{IA} & \mathbf{R}_I \end{bmatrix} \tag{9}$$

Here $\mathbf{R}_A$ and $\mathbf{R}_I$ represent the covariance matrices of $\mathbf{z}_A(n)$ and $\mathbf{z}_I(n)$, respectively, and $\mathbf{R}_{IA} = \mathbf{R}_{AI}^H$ the cross covariance.

Since only the first $M$ elements of $\mathbf{h}_L$ are nonzero, the output power in (7) depends only on the upper left $M$ by $M$ block of $\mathbf{R}_L^{-1}$. Using the block matrix inversion formula, we may rewrite (7) as

$$P(L, M, \mathbf{h}) = \left(\mathbf{h}^H \mathbf{R}_E^{-1}\mathbf{h}\right)^{-1} \tag{10}$$

where

$$\mathbf{R}_E = \mathbf{R}_A - \mathbf{R}_{AI}\mathbf{R}_I^{-1}\mathbf{R}_{IA} \tag{11}$$

$P(L, M, \mathbf{h})$ is maximized over $\mathbf{h}$ for a given $L$ and $M$ by choosing $\mathbf{h}$ as the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_E$. Assuming the largest eigenvalue is not repeated, this channel estimate is unique up to a complex scalar. The corresponding value of the output power is given by the largest eigenvalue of $\mathbf{R}_E$, so we choose $L$ and $M$ to maximize the largest eigenvalue of the matrix $\mathbf{R}_E$.

It is straightforward to show that $\mathbf{R}_E$ is the error covariance matrix associated with the minimum MSE estimate of $\mathbf{z}_A(n)$ from $\mathbf{z}_I(n)$. That is, $\mathbf{R}_E = E\left\{\mathbf{z}_E(n)\mathbf{z}_E^H(n)\right\}$ where $\mathbf{z}_E(n) = \mathbf{z}_A(n) - \mathbf{W}\mathbf{z}_I(n)$ and $\mathbf{W}$ is chosen to minimize the squared error $E\left\{\mathbf{z}_E^H(n)\mathbf{z}_E(n)\right\}$. In the absence of noise (or at high interference to noise ratios), $\mathbf{W}$ can completely cancel the interference provided the rank of the interference covariance matrix is less than or equal to $N-M$ [7]. The rank of the interference covariance matrix depends on the nature of the interference channels. If they are deterministic, then each interferer makes a rank one contribution to the interference covariance matrix and up to $N-M$ interferers can be canceled.

Note that in general, if $L$ and $M$ are incorrect, then the desired user contributes to $\mathbf{z}_I(n)$ and $\mathbf{W}$ will exploit the correlation between the desired user components of $\mathbf{z}_I(n)$ and $\mathbf{z}_A(n)$ to cancel the desired user component of $\mathbf{z}_A(n)$. Hence $P(L, M, \mathbf{h})$ is maximum when $L$ and $M$ are correct.

### 3.2. Quadratically Constrained Approach

Sayeed and Aazhang [2] assume a noncoherent channel model based on the second order statistics $\mathbf{R}_h = E\left\{\mathbf{h}(n)\mathbf{h}^H(n)\right\}$.

We shall assume that an estimate, $\hat{\mathbf{R}}_h$, of $\mathbf{R}_h$ is available and that $\hat{\mathbf{R}}_h$ is nonsingular. It is straightforward to show that the interference resistant quadratic processor in [2] is always a rank one Hermitian matrix. Hence, their constrained optimization problem is equivalent to the quadratically constrained minimum MSE problem

$$\min_{\mathbf{w}} E\left\{\left|\mathbf{w}^H \mathbf{z}_L(n)\right|^2\right\} \quad \text{subject to} \quad \mathbf{w}^H \hat{\mathbf{R}}_{h_L} \mathbf{w} = 1 \qquad (12)$$

where we have defined the $N$ by $N$ matrix

$$\hat{\mathbf{R}}_{h_L} = \begin{bmatrix} \hat{\mathbf{R}}_h & \mathbf{0}_{M,N-M} \\ \mathbf{0}_{N-M,M} & \mathbf{0}_{N-M,N-M} \end{bmatrix} \qquad (13)$$

Partition $\mathbf{w}$ as $\mathbf{w} = \begin{bmatrix} \mathbf{w}_A^T & \mathbf{w}_I^T \end{bmatrix}^T$ so that (12) may be rewritten as

$$\min_{\mathbf{w}} \quad \begin{bmatrix} \mathbf{w}_A^H & \mathbf{w}_I^H \end{bmatrix} \begin{bmatrix} \mathbf{R}_A & \mathbf{R}_{AI} \\ \mathbf{R}_{IA} & \mathbf{R}_I \end{bmatrix} \begin{bmatrix} \mathbf{w}_A \\ \mathbf{w}_I \end{bmatrix}$$

$$\text{subject to} \quad \mathbf{w}_A^H \hat{\mathbf{R}}_h \mathbf{w}_A = 1 \qquad (14)$$

The technique of completing the square can be used to show that

$$\mathbf{w}_I = -\mathbf{R}_I^{-1} \mathbf{R}_{IA} \mathbf{w}_A \qquad (15)$$

and that (12) is thus equivalent to

$$\min_{\mathbf{w}_A} \mathbf{w}_A^H \mathbf{R}_E \mathbf{w}_A \quad \text{subject to} \quad \mathbf{w}_A^H \hat{\mathbf{R}}_h \mathbf{w}_A = 1 \qquad (16)$$

The error covariance matrix $\mathbf{R}_E$ also plays a prominent role in this approach to acquisition. Now introduce the whitening transformation $\tilde{\mathbf{w}}_A = \hat{\mathbf{R}}_h^{\frac{1}{2}} \mathbf{w}_A$ to obtain

$$\min_{\tilde{\mathbf{w}}_A} \tilde{\mathbf{w}}_A^H \tilde{\mathbf{R}}_E \tilde{\mathbf{w}}_A \quad \text{subject to} \quad \tilde{\mathbf{w}}_A^H \tilde{\mathbf{w}}_A = 1 \qquad (17)$$

where $\tilde{\mathbf{R}}_E = \hat{\mathbf{R}}_h^{-\frac{H}{2}} \mathbf{R}_E \hat{\mathbf{R}}_h^{-\frac{1}{2}}$ is the whitened error covariance matrix. The minimization problem is solved by choosing $\tilde{\mathbf{w}}_A$ as the eigenvector corresponding to the smallest eigenvalue of $\tilde{\mathbf{R}}_E$. The values of $L$ and $M$ are then chosen to maximize the smallest eigenvalue of $\tilde{\mathbf{R}}_E$.

### 3.3. Comparison

The assumption of uncorrelated channel coefficients, $\hat{\mathbf{R}}_h = \mathbf{I}$, is of particular interest as it allows for direct comparison of quadratic [2] and LCMV [1] approaches. In this case $\tilde{\mathbf{R}}_E = \mathbf{R}_E$ and both methods perform acquisition based on the eigenvalues of the error covariance matrix. The LCMV approach chooses $L$ and $M$ to maximize the largest eigenvalue, while the quadratically constrained approach chooses them to maximize the smallest eigenvalue. Resolution of the apparent contradiction between these methods is obtained by considering the nature of $\mathbf{R}_E$ in light of the respective assumptions on the desired user CMCs.

We may expand $\mathbf{z}_A(n)$ and $\mathbf{z}_I(n)$ as

$$\mathbf{z}_A(n) = b(n)\mathbf{h}_A(n) + \mathbf{i}_A(n) + \mathbf{v}_A(n) \qquad (18)$$

$$\mathbf{z}_I(n) = b(n)\mathbf{h}_I(n) + \mathbf{i}_I(n) + \mathbf{v}_I(n) \qquad (19)$$

where $b(n)$ represents the desired user's bit stream, $\mathbf{h}_A(n)$ and $\mathbf{h}_I(n)$, $\mathbf{i}_A(n)$ and $\mathbf{i}_I(n)$, and $\mathbf{v}_A(n)$ and $\mathbf{v}_I(n)$ represent the active and inactive components of the desired user CMCs, interfering users, and background noise, respectively. If $L$ and $M$ are correctly chosen, then $\mathbf{h}_I(n)$ is zero. Recall that

$$\mathbf{R}_E = E\left\{(\mathbf{z}_A(n) - \mathbf{W}\mathbf{z}_I(n))(\mathbf{z}_A(n) - \mathbf{W}\mathbf{z}_I(n))^H\right\} \qquad (20)$$

where $\mathbf{W}$ is chosen according to

$$\min_{\mathbf{W}} E\left\{||(\mathbf{z}_A(n) - \mathbf{W}\mathbf{z}_I(n))||^2\right\} \qquad (21)$$

For purposes of illustration, we shall assume that the background noise and interference components are insignificant contributors to $\mathbf{R}_E$. This implies that $\mathbf{i}_A(n)$ is nearly perfectly correlated with $\mathbf{i}_I(n)$ and there are sufficient degrees of freedom in $\mathbf{W}$ to estimate $\mathbf{i}_A(n)$ from $\mathbf{i}_I(n)$, and that the background noise is very weak. This is clearly the most favorable condition, since then acquisition is performed based only on the manner in which $\mathbf{R}_E$ depends on the desired user. That is,

$$\mathbf{R}_E \approx E\left\{(\mathbf{h}_A(n) - \mathbf{W}\mathbf{h}_I(n))(\mathbf{h}_A(n) - \mathbf{W}\mathbf{h}_I(n))^H\right\} \qquad (22)$$

If the desired user CMCs $\mathbf{h}(n)$ are deterministic, as assumed in the LCMV approach, then

$$\mathbf{R}_E \approx (\mathbf{h}_A - \mathbf{W}\mathbf{h}_I)(\mathbf{h}_A - \mathbf{W}\mathbf{h}_I)^H \qquad (23)$$

is a rank one matrix whose smallest eigenvalue has multiplicity $M-1$ and is always zero. Hence, the smallest eigenvalue is independent of $L$ and $M$ and the quadraticly constrained method fails to provide acquisition information. The largest eigenvalue of $\mathbf{R}_E$ is equal to

$$\lambda_{max} = ||\mathbf{h}_A - \mathbf{W}\mathbf{h}_I||^2 \qquad (24)$$

If $\mathbf{h}_I \neq \mathbf{0}$, then $b(n)\mathbf{h}_A$ and $b(n)\mathbf{h}_I$ are perfectly correlated and $\mathbf{W}$ chosen according to (21) will cancel some or all of $b(n)\mathbf{h}_A$ from $b(n)\mathbf{h}_I$. Thus, $\lambda_{max} < ||\mathbf{h}_A||^2$. On the other hand, if $\mathbf{h}_I = \mathbf{0}$, then all the channel information is contained in $\mathbf{h}_A$ and $\lambda_{max} = ||\mathbf{h}_A||^2$ is maximized. Note that $\mathbf{h}_I = \mathbf{0}$ when $L$ and $M$ are correct, and for multiple values of $L$ if $M$ is too large. This suggests that the smallest value of $M$ which maximizes the largest eigenvalue should be chosen.

Now suppose the channel is random, and specifically that $\mathbf{R}_h = \mathbf{I}$. In this case $\mathbf{h}_A(n)$ and $\mathbf{h}_I(n)$ are uncorrelated, so $\mathbf{W}$ cannot cancel any of $b(n)\mathbf{h}_A(n)$ using $b(n)\mathbf{h}_I(n)$ and

$$\mathbf{R}_E \approx E\left\{\mathbf{h}_A(n)\mathbf{h}_A^H(n)\right\} \qquad (25)$$

Since $\mathbf{R}_h = \mathbf{I}$, we know that $E\left\{\mathbf{h}_A(n)\mathbf{h}_A^H(n)\right\}$ is a diagonal matrix with ones in the positions corresponding to the desired user's CMCs and zeros elsewhere. Hence, in this case the maximum eigenvalue of $\mathbf{R}_E$ is one provided at least one nonzero desired user CMC is included in $\mathbf{h}_A(n)$ and the LCMV criterion fails to provide unique acquisition information. The smallest eigenvalue is zero, unless all the elements of $\mathbf{h}_A(n)$ contain nonzero desired user CMCs, in which case $\mathbf{R}_E \approx \mathbf{I}$ and the smallest eigenvalue becomes one. Here the smallest eigenvalue is one for multiple values of $L$ whenever $M$ is too small. Hence, the quadratically constrained criterion provides correct acquisition information provided the largest value of $M$ that maximizes the smallest eigenvalue is selected.

We may extrapolate these results slightly to draw the following conclusions. Under ideal conditions of very high effective SINR, use of the minimum eigenvalue of $\mathbf{R}_E$ fails whenever the channel is deterministic or $\mathbf{R}_h$ is singular, while use of the maximum eigenvalue fails whenever $\mathbf{R}_h$ is of rank two or greater.

### 3.4. A Robust Acquisition Criterion

The failure mechanisms of the LCMV and quadratically constrained acquisition methods suggest a new acquisition criterion that is robust to the channel assumptions. We propose performing acquisition by maximizing the trace of the error covariance matrix. That is, we select the minimum value of $M$ and the corresponding $L$ that satisfy

$$\max_{L,M} \operatorname{tr}(\mathbf{R}_E) \tag{26}$$

It is straightforward to show that this gives correct acquisition independent of the rank of $\mathbf{R}_h$ for the cases described in the previous subsection. If the channel is random and characterized by a known $\mathbf{R}_h$, then we maximize the trace of the whitened cost function $\mathbf{R}_h^{-\frac{1}{2}}\mathbf{R}_E\mathbf{R}_h^{-\frac{1}{2}}$.

### 3.5. Channel Estimation

As shown previously, at high SINR and correct $L$ and $M$, $\mathbf{R}_E$ represents an estimate of the desired user CMC covariance matrix $\mathbf{R}_h$. Hence, the eigenvalues of $\mathbf{R}_E$ indicate whether the channel may be considered deterministic or random over the time interval used to estimate $\mathbf{R}_E$. If $\mathbf{R}_E$ is approximately rank one, then the channel is well approximated as deterministic and represented by the dominant eigenvector. If the approximate rank of $\mathbf{R}_E$ is greater than one, then a random channel model is appropriate.

## 4. SIMULATIONS

The ability of each acquisition procedure to correct estimate the timing offset $L$ is illustrated in this section using simulated data. Three interferers of power 30dB stronger than the desired user are present and the background white noise level is varied so that the signal to white noise level (SNR) ranges from -10 to 20 dB. The interferer timing offsets are randomized relative to the desired user over 100 trials at each noise level. The desired and interfering users employ length 63 Gold codes and anti-podal signalling. The multipath spread of the desired user is known to be $M = 4$ while the multipath spreads for the interferers are 2, 3, and 5. In order to illustrate the advantage of the new acquisition criterion we chose the eigenvalues of $\mathbf{R}_h$ to be 1.1, 1.1, 0.1, and 0.1. The eigenvectors of $\mathbf{R}_h$ were chosen as complex random vectors and $\mathbf{R}_h$ is assumed unknown at the receiver. The sample covariance matrix estimate of $\mathbf{R}_L$ is constructed from 70 consecutive length 63 segments of simulated data. Note that this is slightly more than the minimum of 59 data records required for invertibility of the sample covariance matrix estimate of $\mathbf{R}_I$.

The number of correct acquisitions out of 100 trials at each SNR is shown in Fig. 1. As expected, use of the trace criterion yields the best results, attaining better than

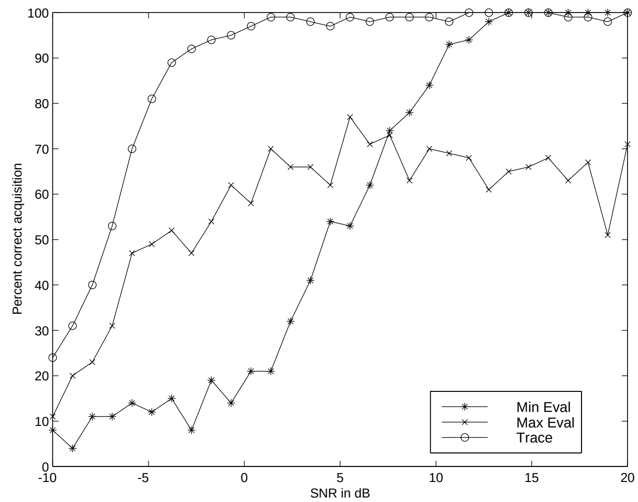


Figure 1: Percent correct acquisition out of 100 trials using the maximum eigenvalue [1], minimum eigenvalue [2] and proposed trace criterion.

95% correct acquisitions for SNR's greater than 0dB. The presence of two equal large eigenvalues in $\mathbf{R}_h$ limits the performance of the method based on the largest eigenvalue even at high SNR since the largest eigenvalue is relatively insensitive to multiple values for $L$. The performance of this method decreases as the number of nearly equal eigenvalues in $\mathbf{R}_h$ increases. Obtaining good acquisition using the smallest eigenvalue requires about 13dB higher SNR than the trace criterion, which is consistent with the observation that the trace of $\mathbf{R}_h$ has 13.8dB more energy than the minimum eigenvalue.

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