Consilient Physical Principles in Wireless Networks

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Abstract

The biggest current challenges in wireless communications are in a network context and are due to the spatially distributed network topology. Recent advances in sensor networks and multi-hop ad hoc communication networks indicate that the key challenges relate to network communication at large scales when the physical size of the network grows with the number of nodes. In this paper, we identify two physical principles that provide insight into network behavior at multiple scales and in different contexts. The two consilient principles are inspired by the physical nature of point-to-point multi-antenna communication over multipath channels – a natural precursor to network communication at smaller scales. We discuss the implications of the two principles in two contexts: i) the impact of signal field statistics on sensor network operation, and ii) viewing large-scale multi-hop cooperative communication as wave propagation in space and time.

1 Introduction

Wireless communication in a network context is radically different from the point-to-point paradigm in Shannon's seminal work. The distributed spatial topology of the network is the key to this paradigm shift and demands a fundamentally different communication perspective. Recent advances provide promising results in different specialized aspects of the puzzle, including *channel-centric* investigations into network transport capacity [1, 2] and capacity of vector broadcast and multiple access channels [3], and *source-centric* studies of distributed source coding techniques [4, 5, 6] and related routing strategies [7] in sensor networks. However, the *source-channel-destination* distinction in Shannon's paradigm gets blurred in a distributed network setting: in a sense *the network is simultaneously the source, the destination and the channel!* This motivates a fresh investigation of the very definition of network channel.

Recent results in network communication underscore that the biggest challenges are related to cooperative communication at *large scales* as the size of the network grows with the number of nodes. In this context, the impact of path loss on communication performance becomes very important [2, 8], an aspect ignored in virtually all other works on network information theory. For example, the most favorable network capacity scaling laws have only been demonstrated

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for small-size networks [2] or under unusually low values of path loss exponent [8]. Similarly, most existing results on sensor networks focus on dense networks in which nodes are concentrated in a small area (see, e.g., [7, 5]). In this paper, we present preliminary ideas on a different perspective on distributed signal processing in sensor networks and multi-hop communication in ad hoc networks. Our perspective is based on consilient physical principles that apply to network communication at *multiple* and *arbitrary* scales. The physical principles have emerged from our significant understanding of point-to-point multi-antenna (MIMO) communication over multipath channels – a natural precursor to network communication at a smaller scale. We discuss the implications of the physical principles in two contexts: i) the impact of signal field characteristics on sensor network operation, and ii) viewing multi-hop cooperative communication in an ad hoc network as controlled space-time wave propagation. These two cases also provide a connection between the source-centric approach in sensor networks to the channel-centric view of communication in ad hoc networks.

2 Two Consilient Physical Principles

CP1. Node $\longleftrightarrow$ scatterer correspondence. This principle is related to an intriguing connection between point-to-point wireless communication via *intermediate scattering objects* and source-destination network communication via *intermediate nodes*. The basic idea is illustrated in Fig. 1. In both cases, the point-to-point channel or the source-destination channel is effectively a time-varying multipath channel in temporal and spectral dimensions. When augmented with the spatial dimension, via antenna arrays in point-to-point links and virtual arrays of nodes in source-destination network links, the effective channel in both cases is a time- and frequency-selective MIMO channel (single-input multiple output (SIMO) or MISO links are also possible). Via this principle, we can leverage our significant understanding of point-to-point multipath communication in the more challenging network setting.

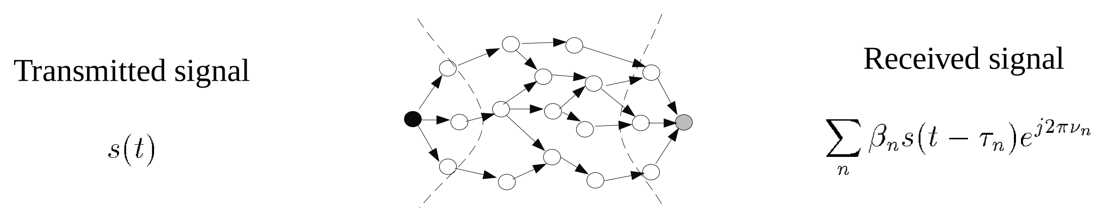

Figure 1: *Illustration of node $\longleftrightarrow$ scatterer correspondence in CP1. In a point-to-point multipath communication link, multiple delayed (τ_n), Doppler (ν_n) shifted and attenuated (β_n) copies of the transmitted signal arrive at the receiver via propagation through intermediate scatterers. In a source-destination multi-hop network link, multiple delayed and Doppler shifted copies of the source signal arrive at the destination via relaying through intermediate nodes. In both cases, the overall communication link can be modeled as a randomly time-varying multipath channel.*

CP2. Multi-dimensional (MD) bandlimited signal fields. MD bandlimited signal fields are lurking everywhere in wireless networks and provide a useful conceptual aid in understanding network behavior at multiple scales. From a channel-centric view, the effective channel underlying the point-to-point wireless link in Fig. 1 can be characterized as a MD bandlimited signal field in temporal, spectral and spatial dimensions. Fig. 2 illustrates the 2D channel associated with Fig. 1 in time and frequency. The channel is a stationary random field in time and frequency and is related to the delay-Doppler representation via a 2D Fourier transform. Fig. 3 illustrates the effective stationary channel in the spatial dimension. From a source-centric view, the signal field in sensor wireless networks can also be modeled as a MD bandlimited signal field in space and time. A key insight is that the overall network behavior is governed by the *interaction* between the source and channel signal fields in time, frequency and space.

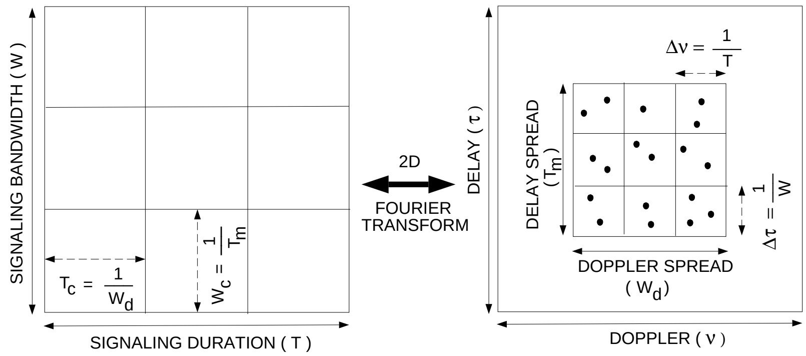

Figure 2: *MD bandlimited signal fields in time and frequency. Representation of the channel in Fig. 1 in time-frequency and delay-Doppler domains. The channel is a 2D stationary random field in time and frequency whose bandwidths are determined by Doppler and delay spreads (W_d and T_m). The channel remains strongly correlated over* time-frequency coherence regions *(TFCRs) of size $T_c \times W_c$ and varies approximately independently over TFCRs. Each delay-Doppler resolution bin (of size $\Delta\tau \times \Delta\nu$) defines a virtual delay-Doppler channel coefficient and only the propagation paths whose delays and Doppler shifts lie in the bin contribute to it.*

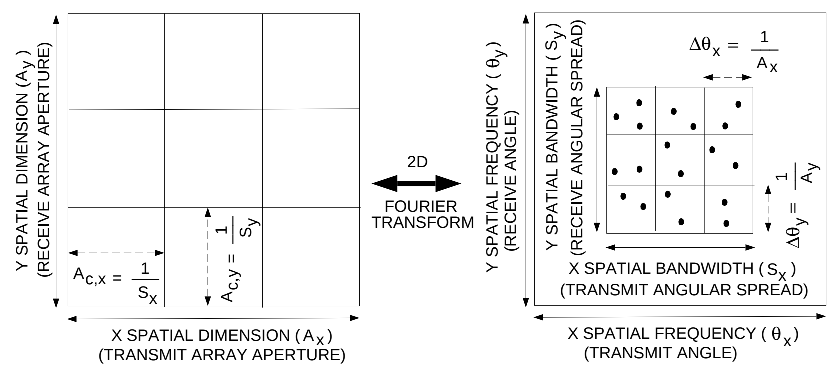

Figure 3: *MD signal fields in space. It represents the 2D stationary spatial random field corresponding to a (virtual) MIMO communication link. The angular spreads of the intermediate (nodes) scatterers determine the spatial bandwidths and the size of the* spatial coherence regions *(SCR's) over which the channel remains strongly correlated. Each angular resolution bin of size $\Delta\theta_x \times \Delta\theta_y$ defines a virtual angular channel coefficient and all propagation paths whose angles lie in the bin contribute to it.*

The two consilient principles have emerged from our significant understanding of wireless communication over multi-antenna time-varying multipath channels in conjunction with more recent investigations into distributed signal processing for sensor networks. In particular, a simple mathematical tool that binds the two principles — *virtual representation of MD bandlimited signal fields* — is at the heart of their discovery [9, 10, 11, 12]. The virtual representation characterizes the stationary bandlimited channel in time, frequency and space (Fig. 2 and Fig. 3) generated by any given distribution of nodes/scatterers (Fig. 1) in terms of *uncorrelated virtual channel coefficients*. Each propagation path via intermediate node/scatterers is associated with unique delay, Doppler shift and spatial angles and the notion of *virtual path partitioning* quantifies the contribution of each propagation path to the degrees of freedom in the channel. The propagation paths (represented by dots) can be partitioned into disjoint delay-Doppler resolution bins, as illustrated in Fig. 2, whose size is determined by the signaling duration and bandwidth. Similarly, the propagation paths can be partitioned into disjoint angular resolution bins, as illustrated in Fig. 3, whose size is determined by the effective spatial apertures (at the transmitter and receiver) associated with the communication link. Each delay-Doppler-angle bin corresponds to a statistically independent virtual channel coefficient, and the number of bins determine the degrees of freedomy and capacity of the channel [11]. Note that the number of bins is proportional to the ”bandwidths” of the signal fields which correspond to the angular, delay and Doppler spreads associated with the nodes/scatterers. The larger the spatial spread of the nodes/scatterers, the larger the bandwidths.

The two consilient principles, along with the virtual representation framework, suggest an approach for studying network communication at multiple scales in time, frequency and space. In the following sections, we discuss the implications of the consilient principles in two contexts: i) signal field modeling in sensor networks, and ii) viewing multi-hop cooperative communication as space-time wave propagation.

3 A Model for the Signal Field in Sensor Networks

The key challenges in sensor networks are tied to two vital operations [13]: 1) efficient information routing between nodes, and 2) collaborative signal processing (CSP) between nodes to extract useful information from the data collected by the sensors. Exchange of sensor information between nodes is necessary due to a variety of reasons, including limited (local) information gathered by each node, variability in operating conditions, and node failure. From a communication and networking viewpoint, sensor networks are similar to ad-hoc multi-hop wireless networks, but there is a key distinction: *the information flow in a sensor network is fundamentally governed by the activity in the physical environment sensed by the nodes.* Furthermore, in view of limited communication and computational capability of nodes, an overarching objective in the design of sensor networks is to exchange the least amount of information between nodes to enable CSP. The notion of bandlimited signal fields in CP2 suggests an approach for modeling the signal field based on the notion of *spatial coherence regions (SCR's)* that captures the salient second-order statistical characteristics of a broad range of sources. In this section we briefly discuss such a modeling approach and its implications on the interplay between information processing and routing; see [14] for a detailed discussion.

3.1 Underlying Assumptions on Signal Statistics

Each signal source corresponds to a space-time signal $s(x, y, t)$ as a function of the spatial coordinates (x, y) and time t. The network nodes sample $s(x, y, t)$ in space and time. Consider a spatial region of interest, $\mathcal{R} = D_x \times D_y = [-D_x/2, D_x/2] \times [-D_y/2, D_y/2]$ associated with a network query involving a single source. We assume that $s(x, y, t)$ is a zero-mean complex circular Gaussian stationary field in the spatial and temporal dimensions

$$s(x,y,t) = \int_{-B_x/2}^{B_x/2}\int_{B_y/2}^{B_y/2}\int_{-B/2}^{B/2} \phi_s(\nu_x,\nu_y,f) e^{j2\pi f_x x} e^{j2\pi f_y y} e^{j2\pi f t} df_x df_y df \qquad (1)$$

where $\phi(f_x, f_y, f)$ denotes the underlying spectral representation which satisfies

$$\mathrm{E}[\phi_s(f_x,f_y,f)\phi_s^*(f_x',f_y',f')] = \Phi_s(f_x,f_y,f)\delta(f_x - f_x')\delta(f_y - f_y')\delta(f - f') \qquad (2)$$

where $\Phi_s(f_x, f_y, f) \geq 0$ represents the power spectral density (PSD) of the field, B_x and B_y represent the spatial signal bandwidths, and B the temporal bandwidth.

3.2 Approximate Modeling Via Spatial Coherence Regions

We propose an approximate signal model, based on *spatial coherence regions (SCR's)* illustrated in Fig. 4(a), that captures the *scales of signal variation* in the spatial coordinates. To a first approximation, the spatial scales of variation in $s(x, y, t)$ are determined by the bandwidths B_x and B_y — the larger the bandwidths, the faster the signal variation in the corresponding dimension. The spatial bandwidths induce *coherence distances*, $D_{c,x} = 1/B_x$ and $D_{c,y} = 1/B_y$ over which the signal remains strongly correlated in the x and y dimensions, respectively. Thus, as illustrated in Fig. 4(a), we partition $\mathcal{R}$ into disjoint SCR's, $\{\mathcal{R}_{m,n}\}$, of size $D_{c,x} \times D_{c,y}$ over which the signal remains strongly correlated. On the other hand, the signal is approximately uncorrelated in distinct SCR's [14].

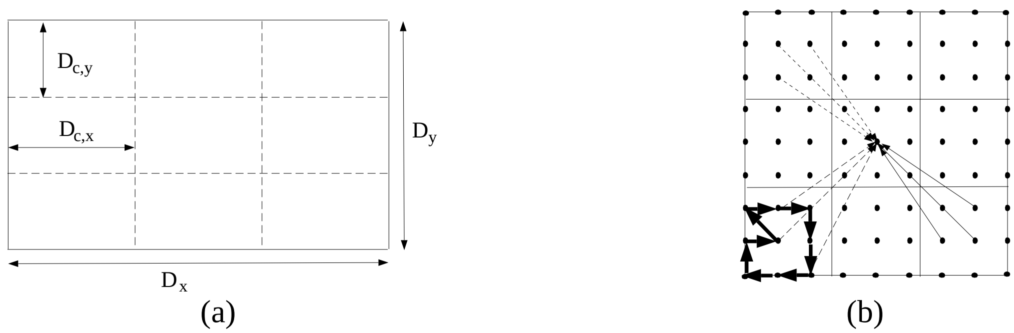

Figure 4: *(a)* **Spatial coherence regions (SCR's)** *over which $s(x, y, t)$ remains strongly correlated. The region $\mathcal{R} = D_x \times D_y$ is partitioned into SCR's, $\{\mathcal{R}_{m,n}\}$, of size $D_{c,x} \times D_{c,y}$. (b) Implications of the PWC model for the design of sensor networks. The big square represents the query region $\mathcal{R}$, and the smaller squares depict the SCR's. Joint processing of high-bandwidth feature-level data is limited to within SCR's, and generates information at the low-bandwidth symbol level. The independent symbols from different SCR's can then be communicated to the destination node/region for final processing. Each SCR represents a single spatial degree of freedom. Node measurements in each SCR may be averaged to improve SNR_{meas}, and the nodes in each SCR may collaborate as a virtual node array to transport the symbol-level information to the destination.*

Specifically, we use a *piece-wise constant (PWC) approximation* of the stationary signal commensurate with the SCR's

$$s_{pwc}(x, y, t) = \sum_{m=-\tilde{N}_x}^{\tilde{N}_x} \sum_{n=-\tilde{N}_y}^{\tilde{N}_y} s_{m,n}(t) I_{D_{c,x}}(x - mD_{c,x}) I_{D_{c,y}}(y - nD_{c,y}) \qquad (3)$$

where $I_X(x)$ denotes the indicator function of the set X, $N_x = D_x/D_{c,x} = 2\tilde{N}_x + 1$, and $N_y = D_y/D_{c,y} = 2\tilde{N}_y + 1$. The PWC signal $s_{pwc}(x, y, t)$ is the projection of $s(x, y, t)$ onto the $N_s = N_x N_y$-dimensional spatial subspace spanned by the orthogonal basis functions $\{u_{m,n}(x, y) = I_{D_{c,x}}(x - mD_{c,x}) I_{D_{c,y}}(y - nD_{c,y})\}$. The N_s temporal processes $\{s_{m,n}(t)\}$ constitute the spatial signal average in the corresponding SCR's

$$s_{m,n}(t) = \frac{1}{D_{c,x} D_{c,y}} \int_{(m-1/2)D_{c,x}}^{(m+1/2)D_{c,x}} \int_{(n-1/2)D_{c,y}}^{(n+1/2)D_{c,y}} s(x, y, t) dx dy. \qquad (4)$$

Note that the *distance-bandwidth (DB) products* $N_x = D_x B_x$ and $N_y = D_y B_y$ are analogous to the time-bandwidth product TB of the space of signals of duration T and bandwidth B.

Spatial degrees of freedom in the signal field. We assume that $\{s_{m,n}(t)\}$ are *perfectly uncorrelated* across distinct SCR's. This assumption can be justified by the fact that most of the spatial correlation in $s(x, y, t)$ is limited to within each SCR, and the residual correlation across SCR's is primarily limited to adjacent SCR's for smooth signal spectra [14]. It can be shown that *there are approximately* $N_s = (D_x B_x)(D_y B_y)$ *independent spatial degrees of freedom in* $s(x, y, t)$ *over* $\mathcal{R}$ *which are preserved by* $s_{pwc}(x, y, t)$.[1]

3.3 Implications of PWC Signal Modeling

Consider a network query involving sensor measurements in a region $\mathcal{R} = D_x \times D_y$ of size $A = D_x D_y \text{ m}^2$. We assume that there are N uniformly distributed nodes in $\mathcal{R}$. The size of each SCR is $A_c = D_{c,x} D_{c,y} = 1/B_x B_y \text{ m}^2$. Thus, there are $N_s = A/A_c = D_x B_x D_y B_y$ SCR's in $\mathcal{R}$ and there are $N_c = N/N_s$ nodes in each SCR.

Space-Time Sampling of the Signal Field. To extract all information about the signal field in $\mathcal{R}$, there should be at least one node in each SCR; that is, $N_c \geq 1$. $N_c = 1$ corresponds to Nyquist sampling ($B_x B_y$ nodes per unit area). If the sensor measurements are noise-free, $N_c = 1$ is sufficient. However, in the presence of noise, additional node measurements in each SCR are advantageous for improving the measurement signal-to-noise ratio, SNR_{meas}. [14].

Interplay between Information Processing and Information Routing. Temporal measurements at each node are often processed in blocks of samples for the application at hand. Information can be exchanged between nodes at two basic levels of abstraction: *feature* or *symbol* level. A *feature* represents a lower-dimensional data representation (e.g. a lower dimensional transform of data block) that contains the relevant signal information. A *symbol* represents a compressed version of the feature vector; for example, a quantized representation in the case of compression or a set of local decisions (soft or hard) in the case of decision making. As argued in [14], the PWC model suggests a hierarchical structure for information exchange between nodes that is naturally suited to the communication constraints of the network (see Fig. 4(b)): *high-bandwidth feature-level exchange is confined to spatially local nodes within each SCR, whereas low-bandwidth symbol-level exchange is sufficient across spatially distant SCR's.* In other words, PWC signal modeling suggests a *fixed matched interface* between CSP and information routing. For example, as suggested in [7], joint compression and routing could be exploited in each SCR.

Distributed Detection and Classification. Detection and classification of objects moving through the sensor field is an important application of sensor networks, e.g., detection and classification of vehicles based on acoustic measurements [16]. There are two main sources of error in distributed detection and classification: i) the noise in sensor measurements, and ii) the inherent variability in the source signal. In [17, 18], we have shown that the structure on information exchange suggested by the PWC model, illustrated in Fig. 4(b), naturally combats the two sources of error. First, feature vectors from a subset of nodes in each SCR can be averaged to yield an effective feature vector at a higher SNR_{meas}. Second, the statistically independent local node decisions (hard or soft, based on the effective feature) from the N_s SCR's are combined to combat the statistical variability in the signal. Numerical results in [18] indicate that a moderate number of relatively unreliable[2] local node decisions (hard or soft) from different SCR's can be combined at the manager node (even via noisy communication links) to yield remarkably reliable[3] final decisions.

Information Rate of the Signal Field and Relation to Network Transport Capacity. It can be shown that for a large number of SCR's (N_s), the differential entropy rate of the signal

[1] We note that similar approximations are widely used in the analysis of randomly time-varying communication channels in the guise of *block fading models* (see, e.g., [15]).

[2] With error probabilities as high as 0.2 or 0.3

[3] With an error probability of 0.01 or smaller.

field, or the minimum bit rate required to compress the signal field at a prescribed distortion, is $\mathcal{O}(N_s B)$ bits/s [14]. Can we transport this data to different parts in $\mathcal{R}$? Recent results in network information theory show that the transport capacity of a network of N nodes grows as $\mathcal{O}(\sqrt{N})$ (bit-meters/s) [1]. Suppose that $D_x = D_y$. Then, there are $\mathcal{O}(\sqrt{N})$ nodes in each spatial dimension. There are two ways in which the network can scale. In the case of *dense scaling* in which the area of $\mathcal{R}$ remains fixed, the information rate of the signal field remains constant at $\mathcal{O}(N_s B) = \mathcal{O}(D_x B_x D_y B_y B)$, and thus the network should be able to transport it for sufficiently large N (this case is also discussed in [7]). The more interesting case is that of *expansive scaling* when the both the area of $\mathcal{R}$ and N_s increase as $\mathcal{O}(N)$ (constant node spacing). In this case, the $\mathcal{O}(\sqrt{N})$ network capacity will not be able to transport the signal field information. However, recent results (see e.g., [2, 8]) indicate that cooperative multi-hop communication may yield more favorable capacity scaling that is sufficient even in this challenging large-scale scenario.

4 Multi-hop Communication as Wave Propagation

From a communication perspective, a sensor network, just like an ad hoc wireless network, is a frothing mix of simultaneous virtual SISO, SIMO, MISO and MIMO links between sets of source and destination nodes. The *spatial scale* of a communication link primarily determines the characteristics of the underlying channel in time, frequency and space. Large-scale (multi-hop) links exhibit larger angular and delay spreads due to larger number of intermediate nodes, whereas small-scale (single- or few-hop) links exhibit minimal dispersion. An important implication is that we have the ability to create the effective channel by choosing the *number* and *spatial distribution* of intermediate nodes, which makes the channel transmission dependent.

In this section, we present some preliminary ideas on a multi-hop communication strategy, inspired by the two consilient principles, that seems promising at large scales. The key idea is to view multi-hop communication as wave propagation that exploits a distinct aspect of the spatial dimension: spatial diversity or power pooling. It well known that the capacity of a single-antenna AWGN channel with bandwidth W scales $C(W) = W\log(1+\mathrm{SNR}/W)$ bits/s whereas the capacity of a MIMO system with N antennas in a rich scattering environment scales as $C(N) \approx N\log(1+\mathrm{SNR})$ bits/s Thus, the capacity saturates with W, whereas it increases linearly with N. This is due to a fundamental difference between the spectral and spatial dimensions: different frequencies are uncoupled (orthogonal), whereas the different antennas are coupled due to spatial scattering.

Can we exploit spatial power pooling in a network context to attain more favorable scaling laws? Since we have a per-node power constraint of P, the ideal capacity scaling potential in a network setting is $N\log(1+N\mathrm{SNR})$, which seems promising. However, there are several difficulties. First, MIMO capacity assumes full cooperation between receivers that is difficult in a large scale network setting. Second, the availability of channel state information (CSI) at the receiver is tenuous. Finally, the effect of path loss exponent must be taken into account.

Basic Idea Behind Wave Propagation. Consider a network with a uniform node spacing d. Let $\alpha > 0$ denote the path loss exponent. The basic idea behind wave propagation is illustrated in Fig. 5. The source node broadcasts its information to its nearest neighbors in the direction of the destination (three nodes in the figure). The three nodes decode and relay the information in the direction of destination in the second wave hop. Due to power pooling from the three nodes, the second wave hop reaches a larger number of nodes, which decode the information and relay it again in the direction of the destination. Consequently, the number of relaying nodes and the distance traveled by the wave increases with each hop. In this way, the source information can be rapidly transported to the destination via cooperation of an increasing number of nodes. This basic idea has also been proposed in [20].

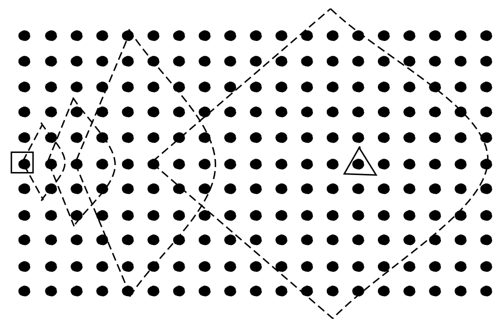

Figure 5: *Source-destination communication via multi-hop wave propagation.*

Impact of Path Loss on Speed of Wave Propagation. We now do an idealized calculation to estimate the impact of α on the speed of wave propagation. We assume that a node can reliably decode the information if the received power exceeds some threshold P_{min}. If the source node transmits at power P, the radius of reliable decoding for the first wave hop is given by $R(1) = (P/P_{min})^{1/\alpha}$. The number of nodes reached in the first hop is given by $N(1) = \pi R(1)^2/2d^2$. For the second and subsequent hops, we have

$$\text{for } i = 2, \cdots, N_{hop} : \; P(i) = N(i-1)P \; , \; R(i) = (P(i)/P_{min})^{1/\alpha} \; , \; N(i) = \pi R(i)^2/2d^2. \quad (5)$$

The above calculation is idealized in the sense that, for estimating the number of nodes reached and distance traveled by i-th wave hop, we are assuming that the total power due to the $N(i-1)$ nodes reached in the previous hop is concentrated at the center of a connectivity disk of radius $R(i)$. Under this idealized assumption, we get for $i = 2, 3, \cdots$

$$\beta(i) = \sum_{k=0}^{i-1}(2/\alpha)^k \; , \; N(i) = N(1)^{\beta(i)} \; , \; P(i) = N(i-1)P \; , \; R(i) = N(1)^{\beta(i-1)/\alpha}R(1). \quad (6)$$

This indicates something remarkable about the speed of wave propagation: $N(i)$ (and hence $P(i)$ and $R(i)$) increases *exponentially* with each hop (i) for $\alpha = 2$, saturates to a value $N(1)^{\alpha/(\alpha-2)}$ for $\alpha > 2$, and increases super-exponentially for $\alpha < 2$. Thus, For $\alpha \leq 2$, the wave accelerates with each each hop at an exponential rate. On the other hand, for the realistic case of $\alpha > 2$, the wave initially accelerates but eventually stabilizes to a constant speed which is determined by P: the larger the P the larger the final wave speed. These different modes of wave propagation are illustrated in Fig. 6.[4]

Impact of Path Loss on Received Power. Consider a receiving (destination) node at the center of a network and assume that all nodes around it are transmitting at power P. There are $\mathcal{O}(k)$ nodes in the k-th ring of radius $R = kd$, $k = 1, 2, \cdots$ around the receiving node. Thus, the received power due to all the nodes in the k-th ring is $P_{rx}(k) \propto (Pk)/(kd)^\alpha = P/(d^\alpha k^{\alpha-1})$, and the total received power due to all nodes within a radius $R = Kd$ is

$$P_{rx}(1:K) \propto \sum_{k=1}^{K} P_{rx}(k) = \frac{P}{d^\alpha} \sum_{k=1}^{K} \frac{1}{k^{\alpha-1}}. \quad (7)$$

[4]Note that the number of bits carried by each wave, coupled with its speed, give the transport capacity of each wave in bit-meters/sec.

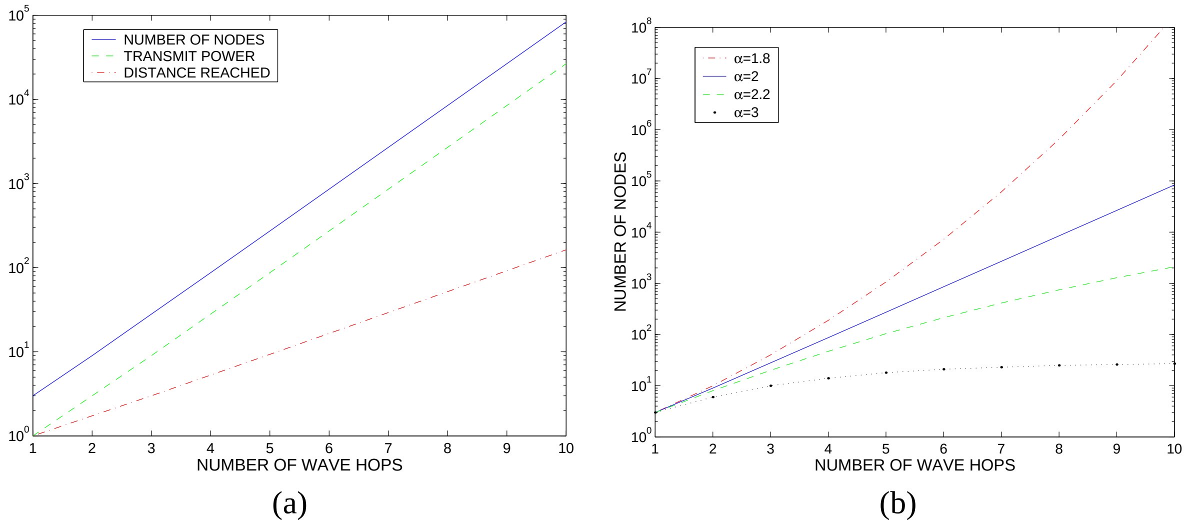

Figure 6: *(a) $N(i)$, $P(i)$ and $R(i)$ as a function of wave hops (i) for $\alpha = 2$. (b) $N(i)$ for $1.8 \leq \alpha \leq 3$.*

The above equation states the received power increases linearly with distance for $\alpha = 1$, logarithmically for $\alpha = 2$ and saturates for $\alpha > 2$ (usually the case in practice).

Wave Speed Versus Multiplexing Tradeoff. The above discussion underscores an important aspect of α (also noted in [1, 2, 8]) in the context of wave propagation. Large α decreases the interference to neighboring nodes (higher spatial reuse) at the cost of wave speed. Smaller α can yield higher wave speeds at the cost of increased interference to other nodes (lesser spatial reuse). The practical case of $\alpha > 2$ seems fortuitous. It says that for any finite P, each node can effectively communicate with only a finite number of nodes and can thus receive and forward a constant amount of information in each time slot. On the other hand, the information from a single source can be communicated to any single destination at a constant speed.

How many simultaneous source-destination flows can be multiplexed via wave propagation? To get an idea, consider a region in which N nodes are cooperating to multiplex a single wave hop of $K \geq 1$ simultaneous source-destination flows. Then, only N/K nodes are available for each flow and thus the distance traveled by the wave for each flow is $((N/K)P/P_{min})^{1/\alpha}$. (Alternatively, only $1/K$-th of the total power, PN, is available for each flow.) Thus, as K increases, the wave speed decreases for each flow. Furthermore, P directly impacts the number of simultaneous flows that can be multiplexed (for any given d and α); e.g., for $\alpha > 2$ each flow uses about $N(1)^{\alpha/(\alpha-2)}$ nodes per wave hop, where $N(1)$ increases with P.

Impact of Antenna Arrays and Scattering. In the context of wave propagation, the key advantage of antenna arrays on each node is to control the spatial pattern of wave propagation. In essence, wave propagation would happen at two spatial scales: the smaller scale of antenna arrays on each node, and the larger scale of virtual node arrays (VNA's) formed by different cooperative nodes. In the absence of antenna arrays, the direction of wave propagation in each hop will be primarily governed by opportunistic beam patterns [21, 20] – the directions in which the signals from N cooperating nodes interfere constructively to reach a distance $(N^2P/P_{min})^{1/\alpha}$ as opposed to the average distance $(NP/P_{min})^{1/\alpha}$. The consilient principles tell us that the *shape* and *degrees of freedom* (the number and location of hot spots) of opportunistic beampatterns depend on both the number of cooperating nodes as well as the size of the region they occupy. The second consilient principle also tells us that the controlled beampatterns due to antenna arrays will exhibit variation in angle on larger scales (due to the smaller array apertures) whereas the opportunistic beampatterns due to VNA's will exhibit variations in angle at smaller scales (due to larger spatial spread of cooperating nodes). Furthermore, the appropriate antenna spacing of the antenna arrays would depend on the average node spacing and the size of the cooperative region to exploit both spatial scales for controlling the wave propagation. The presence of scattering will increase the node-to-node MIMO link capacity and would also facilitate spatial wave dispersion. However, we can only expect rich scattering at sufficiently large node spacings. Furthermore, controlling spatial wave propagation will come at the cost of much higher channel state information.

References

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, pp. 388–404, Mar. 2000.

[2] P. Gupta and P. Kumar, "Towards an information theory of large networks: An achievable rate region," *IEEE Trans. Inform. Th.*, vol. 49, pp. 1877–1894, Aug. 2003.

[3] A. Goldsmith, S. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Selec. Areas Commun. (special issue on MIMO systems)*, vol. 21, pp. 684–702, June 2003.

[4] S. S. Pradhan, J. Kusuma, and K. Ramchandran, "Distributed compression in a dense microsensor network," *IEEE Signal Processing Magazine*, vol. 19, pp. 51–60, Mar. 2002.

[5] D. Marco, E. Duarte-Melo, M. Liu, and D. Neuhoff, "On the many-to-one transport capacity of a dense wireless sensor network and the compressibility of its data," in *Proc. IPSN'03*, pp. 1–16, Apr. 2003.

[6] M. Gastpar and M. Vetterli, "On the capacity of wireless networks," in *Proc. INFOCOM 2002*, June 2002.

[7] A. Scaglione and S. Servetto, "On the interdependence of routing and data compression in multi-hop sensor networks," in *MOBICOM'02*, Sep. 2002.

[8] L. Xie and P. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation," *submitted to the IEEE Trans. Inform. Th.*, Apr. 2002.

[9] A. M. Sayeed and B. Aazhang, "Joint multipath-Doppler diversity in mobile wireless communications," *IEEE Trans. Commun.*, vol. 47, pp. 123–132, Jan. 1999.

[10] A. M. Sayeed, "Deconstructing multi-antenna fading channels," *IEEE Trans. Signal Processing*, vol. 50, pp. 2563–2579, Oct. 2002.

[11] A. M. Sayeed and V. Veeravalli, "The essential degrees of freedom in space-time fading channels," in *Proc. 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'02)*, pp. 1512–1516, Sep. 2002.

[12] V. Raghavan and A. M. Sayeed, "Power laws in the eigenvalue distribution of MIMO wireless channels," in *Proc. 40th Annual Allerton Conference on Communication, Control and Computing*, Oct. 2002.

[13] "Special issue on collaborative signal and information processing in microsensor networks," in *IEEE Signal Processing Magazine*, (S. Kumar and F. Zhao and D. Shepherd (eds.), Mar. 2002.

[14] A. Sayeed, "A statistical signal modeling framework for integrated design of wireless sensor networks," *submitted to the IEEE Signal Processing Magazine*, Oct. 2003.

[15] J. G. Proakis, *Digitial Communications*. New York: McGraw Hill, 3rd ed., 1995.

[16] D. Li, K. Wong, Y. Hu, and A. Sayeed, "Detection, classification, tracking of targets in micro-sensor networks," in *IEEE Signal Processing Magazine*, pp. 17–29, Mar. 2002.

[17] A. D'Costa and A. M. Sayeed, "Collaborative signal processing for distributed classification in sensor networks," in *Proc. IPSN'03)*, pp. 193–208, Apr. 2003.

[18] A. D'Costa, V. Ramachandran, and A. Sayeed, "Distributed classification of gaussian space-time sources in wireless sensor networks," *submitted to the IEEE J. Selec. Areas. Commun.*, July 2003.

[19] E. Telatar, "Capacity of multi-antenna Gaussian channels," *AT& T-Bell Labs Internal Tech. Memo.*, June 1995.

[20] A. Scaglione and Y. Hong, "Opportunistic large arrays: Cooperative transmission in wireless multi-hop ad-hoc networks to reach far distances," *IEEE Trans. Signal Processing*, vol. 51, pp. 2082–2092, Aug. 2003.

[21] P. Viswanath, , D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1277–1294, June 2002.