Low Complexity Space-Time Multiuser Detectors

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Abstract— **A design framework for space-time multiuser detectors is introduced based on the recently introduced notion of canonical space-time coordinates (CSTC). The CSTC system employs a fixed set of basis signals which eliminates the need for estimating direction of arrivals (DOAs) and delays of the channel, thereby allowing significant reduction in receiver complexity. The proposed framework is based on a *hybrid* structure for multiuser detection which combines the power of *centralized* processing and the flexibility of *decentralized* reception. In the first stage, a subset of users whose spreading codes are assumed known are suppressed in a centralized fashion. The remaining interference is suppressed via the second stage in an adaptive decentralized mode. The hybrid scheme offers a flexible design framework by offering lower complexity compared to fully centralized receivers and improved adaptive performance compared to fully decentralized schemes.**

I. Introduction

The use of antenna arrays for enhancing the capacity and quality of multiuser wireless communications has spurred significant interest in space-time signal processing techniques [1]. Most existing space-time receivers employ "ideal" processing matched to all the dominant multipaths and corresponding DOAs. In addition to suffering from high computational complexity in dense multipath environments, such receivers rely heavily on accurate *nonlinear* estimation of the delay and DOA parameters of dominant scatterers. In [2], [3] we introduced a canonical characterization of the received signal in terms of the essential degrees of freedom in the channel that are observable at the receiver. These degrees of freedom are captured by a *fixed* basis corresponding to certain *discrete* multipath delays and DOAs of the signaling waveform. The received signal is mapped onto this basis by sampling the outputs of a space-time matched filter bank. This inherently eliminates the need for estimating arbitrary delays and DOAs, and instead only requires estimates of the channel coefficients, which appear *linearly* in the received signal.

The canonical space-time coordinates (CSTC) induced by the underlying basis provide a natural framework for MAI suppression and diversity processing as the desired user is concentrated in a subset of coordinates that depend on its delay and angle spreads. This fact is exploited in [4] to design a *decentralized* linear MAI-resistant detector that does not require knowledge of the interfering users' signaling waveforms. Decentralized receivers are attractive for mobile units since requiring knowledge of all users' signaling waveforms is not feasible. This knowledge is readily available at the base station and *centralized* receivers that jointly process signals from all the active users are implementable at the base station. Space-time centralized receivers can be developed using the fact that the signal from each user inherits a basis representation induced by CSTC of the corresponding user [5], [4].

Centralized and decentralized receivers admit quite different implementations and offer different complexity and performance trade-offs. Given estimates of required channel parameters, centralized receivers use the matched-filter outputs of all users and, in principle, admit a one-shot implementation. Since all the users are jointly processed, the complexity of centralized receivers is relatively high. Decentralized multiuser receivers, on the other hand, only require knowledge of the signature and channel coefficients of the desired user. Consequently, they require *adaptive* implementation due to the lack of knowledge of codes of interfering users. The linear combiner length required to reliably suppress MAI may be large if the number of the interferers is large. This is problematic since the convergence rate of adaptive algorithms generally worsens as the combiner length increases [6]. A centralized detector does not require adaptation although it may need to adaptively estimate channel coefficients or statistics of all the users. The need for channel estimates of all the users further increases the complexity of centralized receivers.

Motivated by these issues, we propose a two-stage *hybrid* centralized-decentralized multiuser detector structure that combines the attractive features of the two approaches. In the first stage, the signaling waveforms of a subset of interfering users are used in a *centralized* fashion to suppress the corresponding MAI subspace. The remaining MAI is suppressed adaptively using a *decentralized* scheme. The hybrid receiver has lower complexity than a fully centralized receiver. Furthermore, it reduces the number of required adaptive combiner taps in the decentralized stage and can deliver improved performance compared to a fully decentralized receiver.

In Section II, the CSTC multiuser model is reviewed. A slow fading channel is assumed for simplicity. A formulation of centralized and decentralized space-time detectors is presented in Section III, followed by the development of the hybrid structure in Section IV. Performance comparison in terms of signal-to-interference-and-noise-ratio (SINR) of various detector structures is provided in Section V. Illustrative examples are given in Section VI. Section VII summarizes the results presented in this paper.

II. The CSTC Representation

Consider a frequency-selective slow fading channel with K users. The baseband signal received at a R-element array within one signaling interval $[0, T)$ can be written as

$$\mathbf{r}(t) = \sum_{k=1}^{K} b_k \mathbf{s}_k(t-\tau_k) + \mathbf{n}(t),$$
$$\mathbf{s}_k(t) = \int_{S_k^-}^{S_k^+} \int_0^{T_k} H_k(\phi,\tau)\mathbf{a}(\phi) q_k(t-\tau) d\tau d\phi \quad (1)$$

where b_k and τ_k are the symbol and delay of the k-th user, and $\mathbf{a}(\phi)$ is the array response vector for direction-of-arrival (DOA) ϕ. $H_k(\phi, \tau)$ is the angle-dependent impulse response of the fading channel, $q_k(t)$ is the signaling waveform, and $[S_k^-, S_k^+]$ and $[0, T_k]$ are the angular and delay spread for the k-th user, respectively. Here we assume $\{b_k\} \in \{-1, +1\}$ are iid equiprobable random variables and the noise $\mathbf{n}(t)$ is a zero-mean complex Gaussian noise vector with $E[\mathbf{n}(t)\mathbf{n}^H(t')] = \sigma^2 \mathbf{I}_R \delta(t-t')$,[1] independent of $\{b_k\}$. Due to the essentially bandlimited nature of $q_k(t)$, the k-th user signal admits a representation [2], [3]

$$\mathbf{s}_k(t-\tau_k) \approx \sum_{p=P_k^-}^{P_k^+} \sum_{l=d_k}^{d_k+L_k} H_{pl}^{(k)} \mathbf{q}_{pl}^{(k)}(t), \quad (2)$$
$$\mathbf{q}_{pl}^{(k)}(t) = \mathbf{a}(\varphi_p) q_k\left(t - \frac{l}{B}\right), \ 0 \le t < T$$

where B is the effective bandwidth, $\{\varphi_p\}_{p=1}^R$, $-\frac{\pi}{2} \le \varphi_1 < \varphi_2 < \cdots < \varphi_R \le \frac{\pi}{2}$ are chosen such that $\{\mathbf{a}(\varphi_p)\}_{p=1}^R$ are linearly independent. The number of terms in (2) can be determined by $d_k = \lfloor \tau_k B \rfloor$, $L_k = \lceil T_k B \rceil$, $P_k^+ = \min_i\{\varphi_i \ge S_k^+\}$, $P_k^- = \max_i\{\varphi_i \le S_k^-\}$. Without loss of generality, $q_k(t)$ and $\mathbf{a}(\phi)$ are chosen as unit-energy and *user 1* is the desired user.

For simplicity of presentation, we assume that the largest $(\tau_k + T_k) \ll T$. Hence intersymbol-interference (ISI) is negligible and a one-symbol detector suffices.[2] Define the array response and temporal basis matrices as follows

$$\mathbf{A}_k \stackrel{def}{=} \left[\mathbf{a}(\varphi_{P_k^-}), \cdots, \mathbf{a}(\varphi_{P_k^+})\right]$$
$$\boldsymbol{\psi}_k(t) \stackrel{def}{=} \left[q_k\left(t - \frac{d_k}{B}\right), \cdots, q_k\left(t - \frac{d_k+L_k}{B}\right)\right]^T \quad (3)$$

The canonical channel parameter matrix $\mathbf{H}_k$ is defined such that its (p, l) element is $H_{p,l}^{(k)}$. The received signal $\mathbf{r}(t)$ in (1) is sampled at rate B to enable discrete-time processing without loss of information. Define

$$\boldsymbol{\Upsilon} \stackrel{def}{=} \left[\mathbf{r}(0), \mathbf{r}(\frac{1}{B}), \cdots, \mathbf{r}(\frac{N-1}{B})\right] \in \mathcal{C}^{R\times N},$$
$$\mathbf{N} \stackrel{def}{=} \left[\mathbf{n}(0), \mathbf{n}(\frac{1}{B}), \cdots, \mathbf{n}(\frac{N-1}{B})\right],$$
$$\mathbf{Q}_k \stackrel{def}{=} \left[\boldsymbol{\psi}_k(0), \boldsymbol{\psi}_k(\frac{1}{B}), \cdots, \boldsymbol{\psi}_k(\frac{N-1}{B})\right] \quad (4)$$

[1] $\mathbf{I}_R$ denotes the $R \times R$ identity matrix.
[2] Significant asynchronism between users can be accommodated by jointly processing a window of symbols.

Using this notation we can write

$$\boldsymbol{\Upsilon} = \sum_{k=1}^{K} b_k \mathbf{A}_k \mathbf{H}_k \mathbf{Q}_k^H + \mathbf{N}. \quad (5)$$

III. Centralized and Decentralized Receivers

We focus on direct-sequence (DS) CDMA systems for which the CSTC framework is particularly advantageous due to the large time-bandwidth product of the signaling waveforms. In this case $q_k(t) = \sum_{i=0}^{N-1} a_i^{(k)} v(t - iT_c)$ where N is the spreading gain, T_c is the chip duration, and $\{a_i^{(k)}\}$ is the length-N periodic spreading code for user k, and $v(t)$ is a rectangular pulse of duration T_c. We restrict our attention to linear receivers for complexity considerations.

Centralized receivers can be designed along the lines of [5] and require knowledge of the spreading codes, delays and channel spreads of all users. The front-end processing corresponds to correlating the received sampled signal in (5) with the space-time canonical basis of each user

$$\mathbf{Y}_{k'} \stackrel{def}{=} \mathbf{A}_{k'}^H \boldsymbol{\Upsilon} \mathbf{Q}_{k'}$$
$$= \sum_{k=1}^{K} b_k \left(\mathbf{A}_{k'}^H \mathbf{A}_k\right) \mathbf{H}_k \left(\mathbf{Q}_k^H \mathbf{Q}_{k'}\right) + \mathbf{A}_{k'}^H \mathbf{N} \mathbf{Q}_{k'}$$

where $k' = 1, \cdots, K$ denotes the user index. Define $\mathbf{y}_{k'} \stackrel{def}{=} \text{vec}(\mathbf{Y}_{k'}) \in \mathcal{C}^{N_{k'}}$ with $N_{k'} = (L_{k'}+1) \times \left(P_{k'}^+ - P_{k'}^- + 1\right)$ and $\mathbf{h}_k \stackrel{def}{=} \text{vec}(\mathbf{H}_k)$. It can be shown that

$$\mathbf{y}_{k'} = \sum_{k=1}^{K} b_k \mathbf{R}_{k',k} \mathbf{h}_k + \mathbf{w}_{k'},$$
$$\mathbf{R}_{k',k} = \left(\mathbf{Q}_{k'} \otimes \mathbf{A}_{k'}\right)^H \left(\mathbf{Q}_k \otimes \mathbf{A}_k\right) \stackrel{def}{=} \mathbf{B}_{k'}^H \mathbf{B}_k,$$
$$\mathbf{w}_{k'} \sim \mathcal{N}\left[\mathbf{0}, \sigma^2 \mathbf{R}_{k',k'}\right] \quad (6)$$

where $\otimes$ denotes Kronecker product [7]. Combining all $\mathbf{y}_{k'}$ in (6) we get the CSTC of all the users

$$\mathbf{y} \stackrel{def}{=} \left[\mathbf{y}_1^T, \cdots, \mathbf{y}_K^T\right]^T = \mathbf{R}\mathbf{H}\mathbf{b} + \mathbf{w} \in \mathcal{C}^{N_{TOT}}, \quad (7)$$
$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{11} & \cdots & \mathbf{R}_{1K} \\ \vdots & & \vdots \\ \mathbf{R}_{K1} & \cdots & \mathbf{R}_{KK} \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{h}_K \end{bmatrix}$$

where $N_{TOT} = \sum_k N_k$, $\mathbf{b} \stackrel{def}{=} [b_1, \cdots, b_K]^T$ and $\mathbf{w} \sim \mathcal{N}\left[\mathbf{0}, \sigma^2 \mathbf{R}\right]$.

There are two key operations in the linear receiver: MAI suppression and multipath-spatial diversity combing. There are two possible receiver structures depending on the order in which the operations are performed: pre-combiner canceler (diversity combining followed by MAI cancellation) and post-combiner canceler (MAI cancellation followed by diversity combining). The two receiver structures are depicted in Figure 1, corresponding to the canceller matrices $\mathbf{C}_\alpha$ and $\mathbf{C}_\omega$. The output test statistics for the two centralized receiver structures are

$$\begin{aligned}\mathbf{z}_\alpha &= [z_1^\alpha, \cdots, z_K^\alpha]^T = \mathbf{C}_\alpha \alpha, \quad \alpha = [\alpha_1, \cdots, \alpha_K]^T = \mathbf{H}^H \mathbf{y} \\ \mathbf{z}_\omega &= [z_1^\omega, \cdots, z_K^\omega]^T = \mathbf{H}^H \omega, \quad \omega = [\omega_1, \cdots, \omega_K]^T = \mathbf{C}_\omega \mathbf{y}\end{aligned}$$

The detected symbols for K users are given by $\hat{\mathbf{b}} = \text{sgn}(\text{Re}\{\mathbf{z}\})$ in both cases. In the case of minimum mean square error (MMSE) receiver design, the canceler matrices are chosen as [5]

$$\begin{aligned}\mathbf{C}_\alpha &= \arg\min_{\mathbf{X}} \text{E}\,\|\mathbf{X}\alpha - \mathbf{b}\|^2 = \left(\mathbf{H}^H\mathbf{R}\mathbf{H} + \sigma^2\mathbf{I}_K\right)^{-1} \quad (8)\\ \mathbf{C}_\omega &= \mathbf{W} \times \arg\min_{\mathbf{X}} \text{E}\,\|\mathbf{X}\mathbf{y} - \mathbf{H}\mathbf{b}\|^2 = \mathbf{W}\underbrace{\left(\mathbf{R} + \sigma^2\mathbf{\Psi}^{-1}\right)^{-1}}_{\stackrel{def}{=}\mathbf{F}_\omega}\end{aligned}$$

where $\mathbf{\Psi} \stackrel{def}{=} \text{E}\left[\mathbf{H}\mathbf{H}^H\right]$ denotes the average power in the channel coefficients and $\mathbf{W}$ denotes a prewhitening matrix that is given by

$$\mathbf{W} = \begin{bmatrix} \mathbf{K}_{11}^{-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{22}^{-1} & \cdots & \mathbf{0} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{K}_{KK}^{-1} \end{bmatrix} \quad (9)$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{11} & \cdots & \mathbf{K}_{1K} \\ \vdots & & \vdots \\ \mathbf{K}_{K1} & \cdots & \mathbf{K}_{KK} \end{bmatrix} = \mathbf{F}_\omega \mathbf{R} \mathbf{F}_\omega$$

Note that $\mathbf{C}_\alpha$ depends on the channel coefficients whereas $\mathbf{C}_\omega$ only depends on the second-order statistics ($\mathbf{\Psi}$) of $\mathbf{h}_k$. We also note that in the limit of high SNR ($\sigma^2 \to 0$), $\mathbf{C}_\alpha \to \left(\mathbf{H}^H\mathbf{R}\mathbf{H}\right)^{-1}$ and $\mathbf{C}_\omega \to \mathbf{R}^{-1}$, which are decorrelating detectors for pre- and post-combining schemes, respectively [5], with the latter completely independent of the channel. Pre-combining has lower canceler complexity compared to post-combining. However, pre-combining requires estimates of channel coefficients *before* MAI suppression, as opposed to post-combining where the channel coefficients are needed for diversity combining *after* MAI is suppressed. Hence, the pre-combining scheme would likely require more accurate channel estimates.

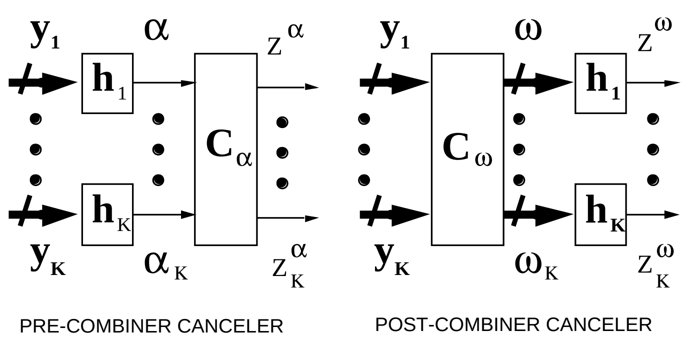

Fig. 1. Centralized receiver stuctures.

Decentralized receivers can be employed when the receiver only knows the desired user's signaling waveform be used [4]. The CSTC system in (2) induces a natural partitioning of the signal space in terms of the CSTCs of the different users. While centralized reception relies on the CSTC of all the users, decentralized receivers operate only on the coordinates of the desired user. The signal corresponding to the desired user is concentrated within a subset of the coordinates, termed *active* coordinates corresponding to the channel spreads encountered by the user. Denoting the overall signal space as $\mathcal{B}$ and active coordinates of desired user as $\mathcal{B}_{A,1}$, we define *inactive* coordinates $\mathcal{B}_{I,1} \stackrel{def}{=} \mathcal{B}\backslash\mathcal{B}_{A,1}$. A basis for $\mathcal{B}_{I,1}$ can be generated from discrete delays and angles of the signaling waveform, as in (2), that lie outside the channel spread. $\mathcal{B}_{A,1}$ captures the desired user's signal and corrupting MAI, whereas $\mathcal{B}_{I,1}$ contains only MAI and no significant component of desired user's signal [2]. Limited MAI suppression can be attained by operating in the low-dimensional active coordinates $\mathcal{B}_{A,1}$. Furthermore, a subset of $\mathcal{B}_{I,1}$ can be incorporated in the detection process to further suppress MAI in the $\mathcal{B}_{A,1}$.

Note that the centralized receiver jointly processes $\{\mathcal{B}_{A,1}, \cdots, \mathcal{B}_{A,K}\}$. The decentralized receiver compensates for the lack of knowledge of $\{\mathcal{B}_{A,2}, \cdots, \mathcal{B}_{A,K}\}$ by incorporating the inactive coordinates $\mathcal{B}_{I,1}$. The decentralized receiver also admits analogues of the pre- and post-combiner structures. In this paper, we focus on the pre-combiner decentralized structure illustrated in Figure 2 where $\mathbf{B}_A = \mathbf{B}_{A,1}$ and $\mathbf{B}_I \subset \mathcal{B}_{I,1}$.

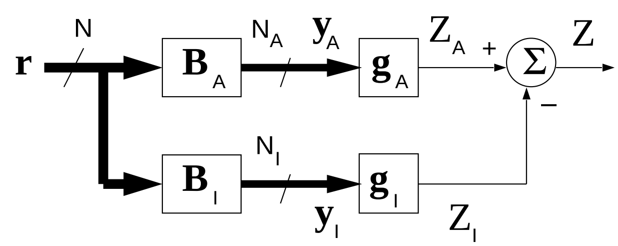

Fig. 2. Decentralized (pre-combiner) receiver structure.

Defining $\mathbf{g}_o = \left[\mathbf{g}_A^H \; -\mathbf{g}_I^H\right]^H$, $\mathbf{y} = \left[\mathbf{y}_A^H \; \mathbf{y}_I^H\right]^H$, $\mathbf{R}_{I,1} = \mathbf{B}_{I,1}^H\mathbf{B}_{A,1}$ and $\mathbf{R}_{yy} = \text{E}\left[\mathbf{y}\mathbf{y}^H\right]$, the optimum linearly constrained minimum variance *(LCMV)* decentralized receiver (Figure 2) for the desired user is [4]

$$\begin{aligned}\mathbf{g}_o &= \arg\min_{\mathbf{g}:\mathbf{g}^H\mathbf{R}_1\hat{\mathbf{h}}_1 = 1} \text{E}[|Z|^2] = \frac{\mathbf{R}_{yy}^{-1}\mathbf{R}_1\hat{\mathbf{h}}_1}{\hat{\mathbf{h}}_1^H\mathbf{R}_1^H\mathbf{R}_{yy}^{-1}\mathbf{R}_1\hat{\mathbf{h}}_1}, \\ \mathbf{R}_1 &\stackrel{def}{=} \left[\mathbf{R}_{11}^H \; \mathbf{R}_{I,1}^H\right]^H \quad (10)\end{aligned}$$

To implement (10), $\mathbf{R}_{yy}$ can be estimated using sample covariance matrix of $\mathbf{y}$ and $\hat{\mathbf{h}}_1$ can be obtained via pilot-aided or blind techniques [8]. Note that $\mathbf{R}_{I,1}$ is generally not zero, hence some signal cancelation may result in the presence of very weak interfering users.

IV. Hybrid Multiuser Receiver

The structure of a two-stage hybrid receiver for user 1 is shown in Figure 3. In the first stage, the CSTCs of user 1 and $K'-1$ other users are jointly processed in a centralized fashion. The remaining MAI from the $K-K'$ other users is suppressed by the second decentralized stage. In this particular set-up, since diversity combining is performed in the decentralized stage, a post-combining scheme must be employed in the centralized stage.

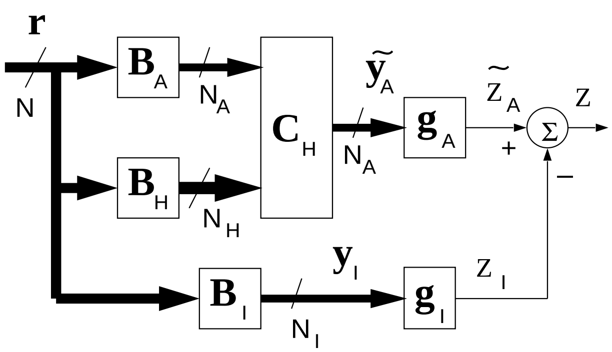

Fig. 3. Hybrid receiver.

For the first centralized stage, define $\mathbf{y} \stackrel{def}{=} \left[\mathbf{y}_A^H \; \mathbf{y}_H^H\right]^H$, $\mathbf{b}_C \stackrel{def}{=} \left[b_1, \cdots, b_{K'}\right]^T$ and

$$\mathbf{R}_C \stackrel{def}{=} \begin{bmatrix} \mathbf{R}_{11} & \cdots & \mathbf{R}_{1K'} \\ \vdots & & \vdots \\ \mathbf{R}_{K'1} & \cdots & \mathbf{R}_{K'K'} \end{bmatrix}, \; \mathbf{H}_C \stackrel{def}{=} \begin{bmatrix} \mathbf{h}_1 & \cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}_{K'} \end{bmatrix} \tag{11}$$

From (8) it follows that the centralized MMSE processor $\mathbf{C}_H$ for the first stage is given by

$$\mathbf{C}_H = \left[\left(\mathbf{R}_C + \sigma^2 \mathbf{\Psi}_C^{-1}\right)^{-1}\right]_{1:N_1,:} \tag{12}$$

where $[\mathbf{M}]_{1:M,:}$ denotes the first M rows of matrix $\mathbf{M}$ and $\mathbf{\Psi}_C = \mathrm{E}\left[\mathbf{H}_C \mathbf{H}_C^H\right]$. The matrix for the decentralized stage follows from (10) by replacing $\mathbf{R}_{yy}$ and $\mathbf{R}_1$ with $\widetilde{\mathbf{R}}_{yy}$ and $\widetilde{\mathbf{R}}_1$, respectively, where $\tilde{\mathbf{y}} \stackrel{def}{=} \left[\tilde{\mathbf{y}_A}^H \; \mathbf{y}_I^H\right]^H$, $\widetilde{\mathbf{R}}_{yy} \stackrel{def}{=} \mathrm{E}\left[\tilde{\mathbf{y}}\tilde{\mathbf{y}}^H\right]$, $\widetilde{\mathbf{R}}_1 \stackrel{def}{=} \left[\mathbf{V}_1^H \;\; \mathbf{R}_{I,1}^H\right]^H$, and

$$\begin{aligned} \mathbf{V} &= \left[\mathbf{V}_1, \cdots, \mathbf{V}_K\right] \stackrel{def}{=} \mathbf{C}_H \left[\mathbf{R}_C \;\; \mathbf{R}_{NC}\right], \\ \mathbf{R}_{NC} &\stackrel{def}{=} \begin{bmatrix} \mathbf{R}_{1,K'+1} & \cdots & \mathbf{R}_{1,K} \\ \vdots & & \vdots \\ \mathbf{R}_{K',K'+1} & \cdots & \mathbf{R}_{K',K} \end{bmatrix}. \end{aligned} \tag{13}$$

As $\sigma^2 \to 0$, $\mathbf{C}_H \to \mathbf{R}_C^{-1}$ which is the decorrelating canceler for the first stage.

The above hybrid scheme only requires channel statistics of $K' < K$ users (1st stage) and estimates for the desired user channel $\hat{\mathbf{h}}_1$ (2nd stage). This results in significantly lower complexity compared to fully centralized reception that may be used in the uplink. The hybrid receiver is also capable of delivering better performance than purely decentralized receivers for the downlink where all users encounter the same channel. Since the first centralized stage suppresses a subset of interfering users, the number of adaptive degrees of freedom required for the second decentralized stage decreases, which gives the hybrid receiver improved convergence behavior compared to fully decentralized receivers.

V. Performance Analysis

The performance of multiuser detectors can be measured in terms of the SINR of the desired user test statistic Z defined in Section III and IV ($Z = z_1^\alpha$ or $Z = z_1^\omega$ for centralized detectors). The desired (first) user test statistic Z can be decomposed into signal, interference, and noise components, that is

$$Z = \zeta_1 b_1 + \sum_{k=2}^{K} \zeta_k b_k + \zeta_n \tag{14}$$

The desired user SINR *given* the channel parameters $\mathbf{H}$ is

$$\mathrm{SINR}(\mathbf{H}) \stackrel{def}{=} \frac{|\zeta_1|^2}{\sum_{k=2}^{K} |\zeta_k|^2 + \mathrm{E}|\zeta_n|^2} \tag{15}$$

It can be shown that for $k = 1, \cdots, K$

Pre-combiner :

$$\begin{aligned} \zeta_k &= \left[\mathbf{C}_\alpha\right]_{1,:} \left[\widehat{\mathbf{H}}^H \mathbf{R} \mathbf{H}\right]_{:,k} \\ \mathrm{E}\left|\zeta_n\right|^2 &= \sigma^2 \left[\mathbf{C}_\alpha\right]_{1,:} \widehat{\mathbf{H}}^H \mathbf{R} \widehat{\mathbf{H}} \left[\mathbf{C}_\alpha\right]_{1,:}^H \\ \mathbf{C}_\alpha &= \left(\widehat{\mathbf{H}}^H \mathbf{R} \widehat{\mathbf{H}} + \sigma^2 \mathbf{I}\right)^{-1} \end{aligned} \tag{16}$$

Post-combiner :

$$\begin{aligned} \zeta_k &= \hat{\mathbf{h}}_1^H \mathbf{K}_{11}^{-1} \left[\sum_{m=1}^{K} \mathbf{F}_{1m}^\omega \mathbf{R}_{mk}\right] \mathbf{h}_k \\ \mathrm{E}\left|\zeta_n\right|^2 &= \sigma^2 \hat{\mathbf{h}}_1^H \mathbf{K}_{11}^{-1} \left[\sum_{m=1}^{K} \sum_{m=1}^{K'} \mathbf{F}_{1m}^\omega \mathbf{R}_{m,m'} \mathbf{F}_{1m'}^{\omega\,H}\right] \mathbf{K}_{11}^{-H} \hat{\mathbf{h}}_1 \\ \mathbf{F}_{1m}^\omega &= \left[\mathbf{F}_\omega\right]_{1:L_1, L_{(m)}+1:L_{(m+1)}} \end{aligned} \tag{17}$$

where $\mathbf{F}$ is given in (8), $L_{(m)} \stackrel{def}{=} \sum_{i=1}^{m} L_i$, and $\widehat{\mathbf{H}}$ denotes the estimate of $\mathbf{H}$. The SINR for the decentralized detector is derived in [4]. Since the hybrid receiver is a serial combination of a post-combiner (without $\mathbf{W}$) and decentralized detector, the SINR computation follows from (17) and the discussion in [4].

VI. Example

We consider a DS-CDMA uplink communication in a multipath Rayleigh fading channel with 6 users. The energy of user k is defined as $e_k \stackrel{def}{=} \mathrm{E}[\mathbf{h}_k^H \mathbf{h}_k]$. The following multipath delays and e_k are used throughout the examples. The SNR of the desired user is 30dB.

User	Delays($\times T_c$)	e_k
1	{ 0,1,2,3,4 }	$e_1 = 0$ dB
2	{ 0,1,2,3 }	$e_2 = dE$ dB
3	{ 0,1,2,3 }	$e_3 = dE$ dB
4	{ 1,2,3 }	$e_4 = dE$ dB
5	{ 1,2,3 }	$e_5 = dE$ dB
6	{ 1,2,3,4 }	$e_6 = dE$ dB

Here dE denotes the interferers' power and T_c is the chip duration (chip-rate sampling is used; $B = 1/T_c$ in (2)). Gold codes with spreading gain of 31 are used for multi-access signature waveforms. The receiver employs only 1 antenna for illustration purposes. The SINR of maximal ratio combiner (MRC) receiver and the various multiuser detectors discussed above is compared. The SINR is averaged over 200 independent channel realizations. In the hybrid receiver, the centralized stage processes users 1,2, and 3. The inactive coordinates in the decentralized stage correspond to the delays $\{6, 7, 8, 9, 10\}T_c$.

Figure 4 depicts the SINR versus dE for various multiuser detectors when MMSE cancelers are used for all centralized stages, both pure centralized and hybrid. The fully centralized pre-combiner canceler attains the highest performance, followed by the decentralized (pre-combiner) detector which uses a subset of inactive coordinates. The centralized post-combiners performs worse than pre-combiners since it performs MAI cancellation in a higher dimensional subspace (prior to diversity combining). The noticeably worse performance of the post-combiner at low interference powers is due to partial cancellation of desired signal. For the same reason, the decentralized detector with active-inactive coordinates slightly outperforms the hybrid detector with active-inactive coordinates in the decentralized stage. The decentralized detector based only on active coordinates is not MAI-resistant since the total number of coordinates (5) is smaller than the number of users (6). However, the hybrid receiver with only active coordinates is MAI-resistant, since users 2 and 3 are suppressed by the centralized section with only 3 other interferers to be suppressed by the decentralized stage.

The poor performance of the post-combiner centralized receiver at low interference power due to signal cancellation can be prevented by treating the interferers with low energy as being absent. Alternatively, use of a decorrelating canceler $\mathbf{C}_H = \mathbf{R}_c^{-1}$ in the centralized stage prevents desired signal cancellation at low interference power. The SINR of various detectors based on a decorrelating centralized stage is shown in Figure 5. The performance of decorrelator detectors is independent of the energy level of all users, although there is up to 4 dB of performance loss at low interference levels due to noise enhancement.

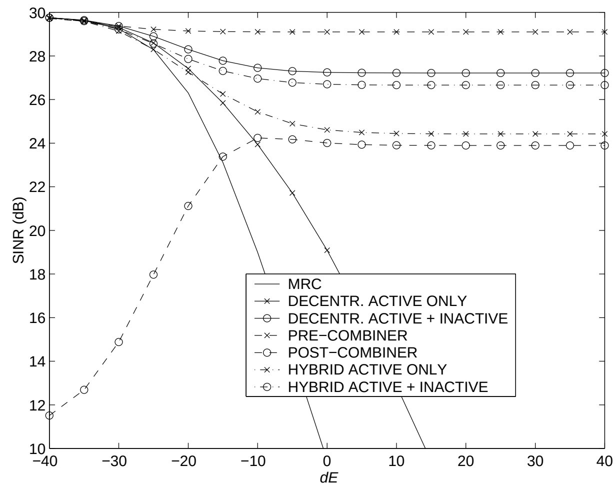

Fig. 4. MMSE canceler in the centralized stage.

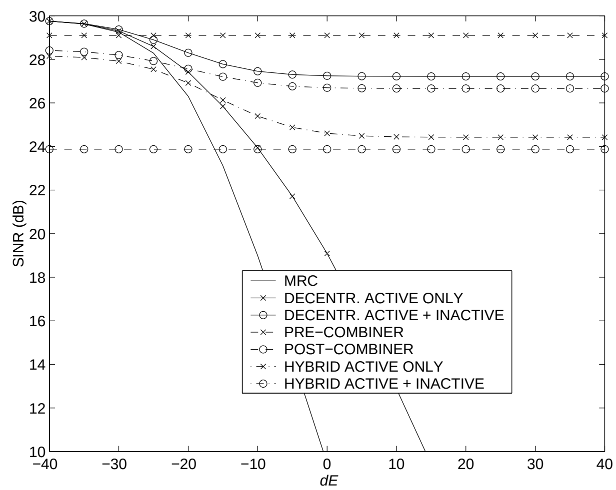

Fig. 5. Decorrelating canceler in the centralized stage.

VII. Discussion

The CSTC framework allows a parameterization of the spatio-temporal channel that is linear with respect to the received signal. Several space-time multiuser detector structures are developed based on the amount of knowledge the receiver has about the interfering users. Centralized and decentralized multiuser receivers form two extremes offering different trade-offs in complexity, performance and implementation. A hybrid centralized-decentralized scheme is proposed to capture the advantages of the two extreme configurations. Its complexity is lower than a fully centralized receiver and promises better convergence properties compared to a fully decentralized (adaptive) receiver. The proposed hybrid schemes provides a flexible design framework for accommodating different performance versus complexity trade-offs.

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