PRECODER CODEBOOK DESIGN FOR LINEAR DISPERSION CODES IN SPATIALLY AND TEMPORALLY CORRELATED MIMO CHANNELS

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ABSTRACT

Despite the popularity and importance of linear dispersion (LD) codes, precoding for LD codes has rarely been investigated. In this paper we propose a method for designing a precoder codebook for LD codes in spatially and temporally correlated channels. Our codebook consists of two parts: a unitary codebook and a diagonal codebook. We propose a systematic algorithm for a unitary codebook design because unitary codebooks are hard to generate using numerical optimization methods. For each column (starting with the first column), we generate a set of vectors in the null space of previous columns. These vectors are generated by skewing i.i.d. unit vector codebooks by means of channel statistics-dependent transformations. A diagonal codebook is generated using a vector quantization technique. We use mutual information as a design criterion throughout this paper. Numerical results illustrate the benefits of matching a codebook to channel statistics and employing adaptive power shaping.

Index Terms— MIMO systems, feedback communication, linear dispersion code, precoding.

1. INTRODUCTION

Adapting transmit signals to channel statistics and/or channel states can significantly improve the capacity and error performance of wireless communication systems. Since the feedback channel is rate-limited, the transmitter should either rely on slowly-varying channel statistics or obtain the limited information on channel states efficiently – in the latter case, the feedback is typically in the form of the index of a chosen beamforming vector or precoding matrix in the codebook agreed a priori by both transmitter and receiver.

Existing works can be categorized by two dimensions: transmission scheme, e.g. beamforming, orthogonal space-time block codes (oSTBCs) and linear dispersion (LD) codes [1]; and the form of feedback, e.g. perfect feedback, statistical feedback, and limited (channel state) feedback. There has been substantial research on feedback schemes for beamforming and orthogonal space-time block codes (refer to [2] for a list of existing work.) However, LD codes have been rarely investigated regarding the exploitation of the feedback resources.

In this paper we propose a precoder codebook design method for LD codes (Threaded Algebraic Space Time codes [3], in particular) in spatially and temporally correlated Rayleigh fading MIMO channels with average mutual information as a design criterion. We divide the codebook design task into the design of a unitary codebook, which is related to (generalized) beam directions or eigendirections, and a diagonal codebook, which specifies the power loading to each direction. There are two key contributions of this work: (i) we propose an algorithm for systematic construction of a unitary codebook matched to spatial channel correlation, and (ii) we prove that the channel statistics-specific transformation in (7) proposed by other researchers (without a rigorous justification) indeed tweaks a codebook of unit vectors for i.i.d. channels into the one whose distribution is the same as the probability density function (pdf) of the normalized MISO channel vector.

Boldface lowercase and uppercase alphabets denote column vectors and matrices, respectively. $||\mathbf{a}|| \triangleq \sqrt{\mathbf{a}^\dagger \mathbf{a}}$ is the ℓ^2-norm. Superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^\dagger$ represent transpose, conjugate, and conjugate transpose, respectively. $\mathbf{A} \sim \mathcal{CN}(\mathbf{M}, \mathbf{R})$ means that $\mathbf{A}$ is complex Gaussian-distributed with mean $\mathbf{M} \triangleq \mathbb{E}[\mathbf{A}]$ and variance $\mathbf{R} \triangleq \mathbb{E}[\text{vec}(\mathbf{A}-\mathbf{M})\text{vec}(\mathbf{A}-\mathbf{M})^\dagger]$, where $\text{vec}(\mathbf{A})$ is the column-stacking operator, i.e. $\text{vec}([\mathbf{a}_1\,\mathbf{a}_2\,\cdots\mathbf{a}_N]) \triangleq [\mathbf{a}_1^T\,\mathbf{a}_2^T\,\cdots\,\mathbf{a}_N^T]^T$. $\otimes$ denotes the Kronecker product and $\text{vand}(\cdot)$ represents the Vandermonde matrix, i.e. $[\text{vand}(\mathbf{a})]_{p,q} = a_p^{q-1}$.

2. SYSTEM MODEL

We consider precoder codebook design for feedback-aided adaptive point-to-point communication systems. We consider narrow-band Rayleigh fading channels with M_t transmit and M_r receive antennas. The channel is assumed to be quasi-static, i.e. it is constant during the transmission of a space-time block of length L, and then transitions into a new state according to a temporal correlation model that appears shortly. Denoting the symbol duration by T_s, the block duration LT_s is typically very small, so the quasi-static assumption is valid. However we should take temporal correlation into account when the feedback delay $\tau \gg LT_s$ is considered. We assume that the channel realization as well as the channel statistics, i.e. channel covariance matrix, is perfectly known at the receiver, but the transmitter knows the channel statistics only. Hence, the receiver selects which precoding matrix the transmitter should use after τ seconds of delay based on the channel realization at time t_0, $\mathbf{H}(t_0)$, and the channel statistics. The index $k \in \{1, 2, \ldots, 2^b\}$ of the selected matrix is conveyed to the transmitter via an error-free feedback channel, where b is the number of available feedback bits.

Let $\mathbf{S}$ denote the $M_t \times L$ transmit signal matrix with $\mathbb{E}[\text{tr}(\mathbf{SS}^\dagger)] = L$, $\mathbf{N}$ the $M_r \times L$ noise matrix following $\mathcal{CN}(\mathbf{0}, \mathbf{I})$, and $\mathbf{W}_k$ the $M_t \times M_t$ precoding matrix chosen by the receiver with $\text{tr}(\mathbf{W}_k\mathbf{W}_k^\dagger) = M_t$. We assume Rayleigh fading channels, that is, the $M_r \times M_t$ channel matrix $\mathbf{H}$ follows $\mathcal{CN}(\mathbf{0}, \mathbf{R})$, where $\mathbf{R} \triangleq \mathbb{E}[\text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^\dagger]$ is the channel covariance matrix. Now the signal received between $t = t_0 + \tau$ and $t = t_0 + \tau + LT_s$ can be written as

$$\mathbf{Y}(t_0+\tau) = \sqrt{E_s}\mathbf{H}(t_0+\tau)\mathbf{W}_k\mathbf{S}(t_0+\tau) + \mathbf{N}(t_0+\tau) \quad (1)$$

where E_s denotes the transmit SNR.

2.1. Temporal Correlation Model

The channel varies over time due to the mobility of the transmitter, receiver and/or scatterers. Hence we are interested in modeling $\mathbf{H}(t_0+\tau)$, the channel state τ seconds after the channel was estimated as $\mathbf{H}(t_0)$ at time t_0. If the delay is close to zero, $\mathbf{H}(t_0+\tau)$ will be close to $\mathbf{H}(t_0)$. If the delay is large, $\mathbf{H}(t_0)$ will bear little information about $\mathbf{H}(t_0+\tau)$ and the only useful information will be the statistics of the channel. Hence the temporal variation is often modeled as [4]

$$\mathbf{H}(t_0+\tau)|\mathbf{H}(t_0) \sim \mathcal{CN}\left(J_0(f_D\tau)\mathbf{H}(t_0), (1-J_0(f_D\tau)^2)\mathbf{R}\right), \tag{2}$$

where $J_0(\cdot)$ is the zero-th order Bessel function of the first kind with $J_0(0)=1$ and $J_0(\infty)=0$, and f_D is the Doppler spread at t_0. It can be easily checked that when $f_D\tau$ is close to zero, $\mathbf{H}(t_0+\tau)|\mathbf{H}(t_0) \sim \mathcal{CN}(\mathbf{H}(t_0), \mathbf{0})$, while as $f_D\tau$ tends to ∞, $\mathbf{H}(t_0+\tau)|\mathbf{H}(t_0) \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$, as desired.

3. PRECODING WITH INFINITE-RATE FEEDBACK

We investigate the infinite-rate feedback case in this section in order to get insight on precoder codebook design, and then consider the finite-rate feedback case in the next section.

Pairwise error probability (PEP) is a widely used design criterion for constructing precoder codebooks for oSTBCs because in this case the worst-case PEP has a simple expression, which represents block error rate reasonably well using the union bound technique. Unfortunately, this is not the case for LD codes where PEP has to be evaluated for thousands or more pairs in order to obtain an approximate block error rate. Hence, mutual information is commonly used as a design criterion [1, 5, 4].

LD codes are written in the form of $\mathbf{S} = \sum_{\ell=1}^{N} \mathbf{A}_\ell a_\ell$, where $\{a_\ell\}_{\ell=1}^{N}$ are i.i.d. complex source symbols taken from a constellation and $\{\mathbf{A}_\ell\}_{\ell=1}^{N}$ is a set of $M_t \times L$ complex dispersion matrices [1]. It is often convenient to write the system equation (1) in a column-stacked form as

$$\text{vec}(\mathbf{Y}) = \sqrt{E_s}\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{W}}\tilde{\mathbf{G}}\mathbf{a} + \text{vec}(\mathbf{N}) \tag{3}$$

where $\tilde{\mathbf{G}} \triangleq [\text{vec}(\mathbf{A}_1) \dots \text{vec}(\mathbf{A}_N)]$, $\mathbf{a} \triangleq [a_1 \dots a_N]^T$, $\boldsymbol{\mathcal{H}} \triangleq \mathbf{I}_L \otimes \mathbf{H}$, $\boldsymbol{\mathcal{W}} \triangleq \mathbf{I}_L \otimes \mathbf{W}$. The $M_t L \times N$ matrices $\tilde{\mathbf{G}}$ and $\mathbf{G} \triangleq \boldsymbol{\mathcal{W}}\tilde{\mathbf{G}}$ are the generator matrix of an LD code and a precoded LD code, respectively. Note that we dropped the subscript k from precoding matrices because in this section we consider the infinite-rate feedback scenario.

When only channel statistics are known at the transmitter, [5] showed that the generator matrix of capacity-achieving LD codes should satisfy the equation $\mathbf{G}\mathbf{G}^\dagger = \mathbf{I}_L \otimes \mathbf{V}_t \boldsymbol{\Lambda}_{\text{opt}} \mathbf{V}_t^\dagger$, where $\mathbf{V}_t$ is the eigenmatrix of the transmit covariance matrix $\mathbf{R}_t \triangleq \mathbb{E}[\mathbf{H}^\dagger \mathbf{H}]$ and $\boldsymbol{\Lambda}_{\text{opt}}$ is a diagonal matrix with nonnegative entries. This extreme case serves as a reference in appraising the gain achieved by feedback.

The other extreme case is when channel realizations are instantaneously known at the transmitter. In this case, it is well known that the optimal input covariance matrix is $\mathbf{V}_H \boldsymbol{\Lambda}_{\text{opt}} \mathbf{V}_H^\dagger$, where $\mathbf{V}_H$ is the right singular matrix of $\mathbf{H} = \mathbf{U}_H \boldsymbol{\Lambda}_H \mathbf{V}_H^\dagger$. Hence the generator matrix should satisfy the equation $\mathbf{G}\mathbf{G}^\dagger = \mathbf{I}_L \otimes \mathbf{V}_H \boldsymbol{\Lambda}_{\text{opt}} \mathbf{V}_H^\dagger$.

Now we consider a general case where the transmitter knows the channel statistics as well as channel realizations but with τ seconds of feedback delay. Applying Jensen's inequality yields the upperbound of average mutual information conditioned on the channel realization $\mathbf{H}(t_0)$ [4]

$$\begin{aligned}\mathcal{I} =& \mathbb{E}[\log\det(\mathbf{I}_{M_t} + E_s \mathbf{Q}\mathbf{H}(t_0+\tau)^\dagger \mathbf{H}(t_0+\tau))] \\ &\le \log\det\left(\mathbf{I}_{M_t} + E_s \mathbf{Q}\boldsymbol{\Phi}(\mathbf{H}(t_0), \mathbf{R}_t)\right),\end{aligned} \tag{4}$$

where $\boldsymbol{\Phi}(\mathbf{H}(t_0), \mathbf{R}_t) \triangleq \mathbb{E}[\mathbf{H}(t_0+\tau)^\dagger \mathbf{H}(t_0+\tau)] = J_0(f_D\tau)^2 \mathbf{H}(t_0)^\dagger \mathbf{H}(t_0) + (1-J_0(f_D\tau)^2)\mathbf{R}_t$ follows from (2). Here the expectations are with respect to the distribution of $\mathbf{H}(t_0+\tau)$ conditioned on $\mathbf{H}(t_0)$. The optimal input covariance $\mathbf{Q}$ that maximizes (4) subject to $\text{tr}(\mathbf{Q}) \le L$ is known to be $\mathbf{Q}_{\text{opt}} = \mathbf{V}_{\boldsymbol{\Phi}} \boldsymbol{\Lambda}_{\text{opt}} \mathbf{V}_{\boldsymbol{\Phi}}^\dagger$, where $\mathbf{V}_{\boldsymbol{\Phi}}$ is the eigenmatrix of $\boldsymbol{\Phi}(\mathbf{H}(t_0), \mathbf{R}_t) = \mathbf{V}_{\boldsymbol{\Phi}} \boldsymbol{\Lambda}_{\boldsymbol{\Phi}} \mathbf{V}_{\boldsymbol{\Phi}}^\dagger$. We can obtain $\boldsymbol{\Lambda}_{\text{opt}}$ via waterfilling as $[\boldsymbol{\Lambda}_{\text{opt}}]_{\ell,\ell} = (\mu - 1/(E_s[\boldsymbol{\Lambda}_{\boldsymbol{\Phi}}]_{\ell,\ell}))^+$, $\ell = 1, 2, \dots, M_t$, where $(\cdot)^+ = \max(\cdot, 0)$ and μ is chosen such that $\mathbf{Q}_{\text{opt}}$ satisfies the trace constraint. Hence the optimal generator matrix should satisfy

$$\mathbf{G}\mathbf{G}^\dagger = \mathbf{I}_L \otimes \mathbf{V}_{\boldsymbol{\Phi}} \boldsymbol{\Lambda}_{\text{opt}} \mathbf{V}_{\boldsymbol{\Phi}}^\dagger. \tag{5}$$

We focus on LD codes that satisfy $\tilde{\mathbf{G}}\tilde{\mathbf{G}}^\dagger = \mathbf{I}_{M_t L}$. It can be easily shown that TAST codes [3] satisfy this property and we use this code in our simulation. Due to this property, we have $\mathbf{G}\mathbf{G}^\dagger = \mathbf{I}_L \otimes \mathbf{W}\mathbf{W}^\dagger$, and by choosing $\mathbf{W}$ appropriately the LD codes can be made near capacity-optimal.

4. PRECODING WITH FINITE-RATE FEEDBACK

We propose an algorithm for designing a finite codebook of precoding matrices for LD codes such that the optimal $\mathbf{G}\mathbf{G}^\dagger$ in (5) can be closely approximated by a precoding matrix in the codebook. Numerical optimization [2] is not well-suited for this task because of the lack of a simple worst-case PEP expression and large number of optimization variables. To elaborate further, write the precoding matrix $\mathbf{W}_k$, $k = 1, \dots, 2^b$, in the singular value decomposition form $\mathbf{W}_k = \tilde{\mathbf{U}}_k \tilde{\mathbf{D}}_k \mathbf{V}_0$. Now $\tilde{\mathbf{D}}_k$ can be specified by $M_t - 1$ real parameters, which is relatively small, but $\tilde{\mathbf{U}}_k$ requires $M_t^2 - M_t$ real parameters, which becomes very large even with a moderate number of antennas [2]. Hence we divide the codebook design task into the design of a unitary codebook $\{\mathbf{U}_n\}_{n=1}^{2^{b_U}}$ and that of a diagonal codebook $\{\mathbf{D}_m\}_{m=1}^{2^{b_D}}$, where b_U and b_D are the number of feedback bits allocated to the unitary codebook and the diagonal codebook, respectively, satisfying $b_U + b_D = b$, the total number of available feedback bits. We then construct the unitary codebook systematically, while generating the diagonal codebook using a vector quantization technique. Any choice of the unitary matrix $\mathbf{V}_0$ does not affect mutual information, but it does affect the error performance. We arbitrarily chose the following structure to allow a single parameter optimization, and this parameter β can be found by an exhaustive search such that it yields the best average PEP for a number of pairs of transmit signal matrices. For $M_t = 4$ and BPSK constellation, the best β for $\mathbf{V}_0 = (1/\sqrt{M_t})\text{vand}([e^{j\beta}\ e^{j(\beta+\pi/2)}\ e^{j(\beta+\pi)}\ e^{j(\beta+3\pi/2)}]^T)$ is found to be 0.97.

4.1. Systematic Unitary Codebook Design

We now present a unitary codebook design method. The key idea is that we can construct a set of unitary matrices by successively filling columns, one column after another, with a set of vectors that are not only orthogonal to previous columns but also matched to the spatial correlation of the channel.

To the best of our knowledge, there is no known algorithm for systematically generating a codebook of *unitary matrices* matched to channel statistics. The idea of skewing an i.i.d. codebook of *beamforming vectors* based on the channel covariance matrix has been

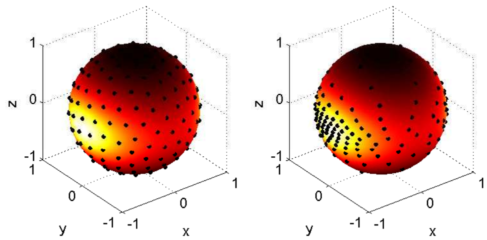

Fig. 1. Effect of the transform in (7) on the distribution of codeword vectors. Dots represent vectors in an i.i.d. unit vector codebook (left sphere) and a transformed codebook (right sphere).

proposed by a few authors, but the proposed skewing method was not justified mathematically. In [6], the authors propose an algorithm for the systematic construction of a codebook of unitary matrices for i.i.d. channels, but that is not easily extendable to correlated channels. For spatially correlated channels the problem is very difficult because the pdf of $\mathbf{V_\Phi}$, which should be the left singular matrix of $\mathbf{W}$ in order to satisfy (5) in the infinite-rate feedback case, is not known. In fact, the pdf of $\mathbf{V_\Phi}$ is an open problem even for the simpler case of $J_0(f_D\tau) = 1$ and we only know that in the limit of the number of receive antennas, the first column of $\mathbf{V_\Phi}$ is close to the dominant statistical eigendirection $\mathbf{v}_{t,1}$ of $\mathbf{R}_t = \mathbf{V}_t\mathbf{\Lambda}_t\mathbf{V}_t^\dagger = \sum_{\ell=1}^{M_t}[\mathbf{\Lambda}_t]_{\ell,\ell}\mathbf{v}_{t,\ell}\mathbf{v}_{t,\ell}^\dagger$ with high probability [7]. When the number of receive antennas is finite, we conjecture that the distribution of $\mathbf{V_\Phi}$ depends on $\mathbf{R}_t$ in a fashion that the first column of $\mathbf{V_\Phi}$ lies around the stronger statistical eigendirections such as $\mathbf{v}_{t,1}$ more often than around the weaker statistical eigendirections such as $\mathbf{v}_{t,M_t}$.

When $M_r = 1$ and $f_D\tau = 0$, we can compute the pdf of $\mathbf{V_\Phi}$. In this case, we can see that $\mathbf{\Phi}(\mathbf{h}(t_0), \mathbf{R}_t)$ is rank 1, and hence the diagonal entries of $\mathbf{\Lambda_\Phi}$ and $\mathbf{\Lambda}_{\text{opt}}$ are zero except for the first diagonal entry. (We used different notation $\mathbf{h}(t_0)$ replacing $\mathbf{H}(t_0)$ because the channel is now a vector, not a matrix.) This implies that $\mathbf{G}\mathbf{G}^\dagger$ in (5) is determined by the dominant eigenvector $\mathbf{v}_{\mathbf{\Phi},1}$ of $\mathbf{\Phi}(\mathbf{h}(t_0), \mathbf{R}_t)$ (other eigenvectors correspond to zero eigenvalues and are irrelevant) and that we only need to design a set of vectors for the first column of $\{\mathbf{U}_n\}_{n=1}^{b_U}$ such that the distribution of those vectors is matched to the distribution of $\mathbf{v}_{\mathbf{\Phi},1}$. Note that $\mathbf{v}_{\mathbf{\Phi},1}$ is just the right singular vector $\tilde{\mathbf{h}}(t_0)^T$ of the $1\times M_t$ channel vector $\mathbf{h}(t_0)^T$ and we can derive the pdf of $\mathbf{v}_{\mathbf{\Phi},1}$. (Due to the page limit, the proofs of the lemmas are not provided.)

Lemma 1 *Denote a MISO channel by* $\mathbf{h}(t_0)^T \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_t)$. *The right singular vector* $\tilde{\mathbf{h}}(t_0)^T$ *of* $\mathbf{h}(t_0)^T$ *is simply* $\mathbf{h}(t_0)^T/||\mathbf{h}(t_0)^T||$. *Then the pdf of* $\tilde{\mathbf{h}}(t_0)$ *is*

$$P(\tilde{\mathbf{h}}(t_0)) = (M_t-1)!/2\pi^{M_t}|\mathbf{R}_t|(\tilde{\mathbf{h}}(t_0)^\dagger\mathbf{R}_t^{-1}\tilde{\mathbf{h}}(t_0))^{M_t}. \quad (6)$$

Interestingly, it is possible to generate a codebook of unit vectors whose distribution is identical to (6) simply by skewing a reference codebook of unit vectors designed for i.i.d. channels. Consider the following transformation which maps an M_t-dimensional vector $\mathbf{c} = [c_1\, c_2\, \ldots\, c_{M_t}]^T$ into $\mathbf{u}$ of the same dimension

$$\mathbf{u} = \sum_{\ell=1}^{M_t} c_\ell[\mathbf{\Lambda}_t]_{\ell,\ell}^{1/2}\mathbf{v}_{t,\ell}/||\sum_{\ell=1}^{M_t} c_\ell[\mathbf{\Lambda}_t]_{\ell,\ell}^{1/2}\mathbf{v}_{t,\ell}||. \quad (7)$$

Recall that $\mathbf{v}_{t,\ell}$ and $[\mathbf{\Lambda}_t]_{\ell,\ell}$ are the ℓ-th eigenvector and eigenvalue of $\mathbf{R}_t = \mathbf{V}_t\mathbf{\Lambda}_t\mathbf{V}_t^\dagger$, respectively. Observe that $\mathbf{v}_{t,\ell}$ functions as a new basis vector and the square root of $[\mathbf{\Lambda}_t]_{\ell,\ell}$ as a weighting factor.

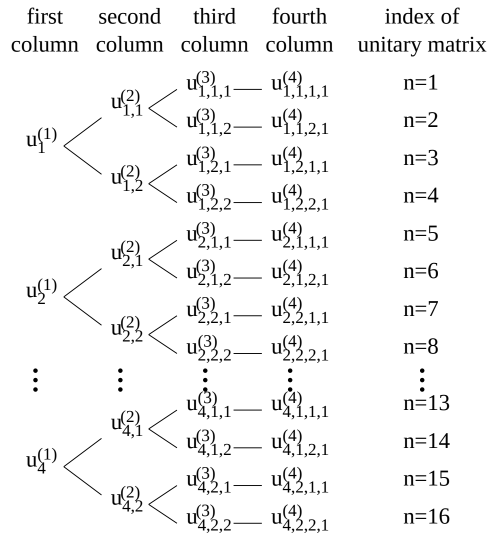

Fig. 2. An example of a unitary codebook of 16 matrices designed with $b_1 = 2$, $b_2 = b_3 = 1$, $b_4 = 0$ bit(s).

Indeed, the next lemma shows that this transformation tweaks an i.i.d. unit vector codebook into a codebook whose distribution is the same as (6). See Fig. 1 for an illustration of the distribution of vectors before and after the transformation.

Lemma 2 *The Jacobian determinant of the transformation in (7), when viewed as a mapping from* $\mathbb{R}^{2M_t}$ *into itself, is* $J = |\mathbf{\Lambda}_t|\,||\mathbf{c}||^{2M_t}(\mathbf{u}^\dagger\mathbf{R}_t^{-1}\mathbf{u})^{M_t}$. *Hence if* $\mathbf{c}$ *is uniformly distributed on the surface of a unit sphere, then the pdf of* $\mathbf{u}$ *is*

$$P(\mathbf{u}) = P(\mathbf{c})/J = 1/A|\mathbf{\Lambda}_t|(\mathbf{u}^\dagger\mathbf{R}_t^{-1}\mathbf{u})^{M_t} \quad (8)$$

where $A = 2\pi^{M_t}/(M_t-1)!$ *is the surface area of a unit sphere.*

Observe that (8) is exactly same as (6). So far we have shown that we can generate a unit vector codebook matched to spatial channel correlation by skewing an i.i.d. unit vector codebook. Building upon this result, we now present a unitary codebook design algorithm. First, note that the columns on the left side of $\{\mathbf{U}_n\}_{n=1}^{2^{b_U}}$ are more important than the ones on the right side. For example, the first column of $\mathbf{U}_n \approx \mathbf{V_\Phi}$ corresponds to the strongest eigenvector of $\mathbf{\Phi}(\mathbf{H}(t_0), \mathbf{R}_t)$, and it is loaded with more power than other columns because it corresponds to the largest eigenvalue $[\mathbf{D}_m]_{1,1}$. Hence we construct a unitary codebook filling column after column allocating more resource (feedback bits) to the left columns. That is, if b_ℓ denotes the number of feedback bits allocated to the ℓ-th column, we choose $\{b_\ell\}_{\ell=1}^{M_t}$ such that $b_1 \geq b_2 \geq \cdots \geq b_{M_t-1} \geq b_{M_t} = 0$ and $\sum_{\ell=1}^{M_t} b_\ell = b_U$. Second, for unitariness, a vector in the ℓ-th column $\mathbf{u}^{(\ell)}$ should be orthogonal to previous columns, $\mathbf{u}^{(1)}, \ldots, \mathbf{u}^{(\ell-1)}$. The following is the procedure for constructing a unitary codebook.

Step 1: We need to generate 2^{b_1} unit vectors that will be used in the first columns of $\{\mathbf{U}_n\}_{n=1}^{2^{b_U}}$. These vectors should represent the first column of the random matrix $\mathbf{V_\Phi}$ reasonably well. As discussed in the beginning of Section 4.1, we expect that the first column of $\mathbf{V_\Phi}$ has a higher pdf along the statistically dominant transmit eigendirections such as $\mathbf{v}_{t,1}$ than the weaker directions such as $\mathbf{v}_{t,M_t}$. Hence we use the transformation in (7) to skew each vector in an M_t-dimensional i.i.d. unit vector codebook $\{\mathbf{c}_n^{(1)}\}_{n=1}^{2^{b_1}}$ to obtain $\{\mathbf{u}_{n_1}^{(1)}\}_{n_1=1}^{2^{b_1}}$.

Step 2: Next we generate 2^{b_2} unit vectors that will be used in the second columns of $\{\mathbf{U}_n\}_{n=1}^{2^{b_U}}$. For unitariness, these vectors should be orthogonal to the first column and hence we should have one unit vector codebook of cardinality 2^{b_2} for each vector in $\{\mathbf{u}_{n_1}^{(1)}\}_{n_1=1}^{2^{b_1}}$. This property makes a unitary codebook look like a tree as illustrated in Fig. 2. Due to the orthogonality constraint, we need to slightly modify (7) as $\mathbf{u}_{n_1,n_2}^{(2)} = \sum_{\ell=1}^{M_t-1}[\mathbf{c}_{n_2}^{(2)}]_\ell[\mathbf{\Lambda}_t^{(2)}]_{\ell,\ell}^{1/2}\mathbf{v}_{t,\ell}^{(2)}/||\sum_{\ell=1}^{M_t-1}[\mathbf{c}_{n_2}^{(2)}]_\ell[\mathbf{\Lambda}_t^{(2)}]_{\ell,\ell}^{1/2}\mathbf{v}_{t,\ell}^{(2)}||$, where $\mathbf{v}_{t,\ell}^{(2)}$ and $[\mathbf{\Lambda}_t^{(2)}]_{\ell,\ell}$ are the ℓ-th eigenvector and eigenvalue of the projected transmit covariance matrix $\mathbf{P}(n_1)\mathbf{R}_t\mathbf{P}(n_1)$. Here $\mathbf{P}(n_1) = \mathbf{I}_{M_t} - \mathbf{u}_{n_1}^{(1)}\mathbf{u}_{n_1}^{(1)\dagger}$ is a matrix projecting a vector onto the null space of $\mathbf{u}_{n_1}^{(1)}$, and $\{\mathbf{c}_n^{(2)}\}_{n=1}^{2^{b_2}}$ is an (M_t-1)-dimensional i.i.d. unit vector codebook. Proceed in a similar fashion until all the columns are provided with a set of vectors.

Working with i.i.d. unit vector codebooks whose first vector is $[1\,0\,\ldots\,0]^T$, we get $\mathbf{U}_1 = \mathbf{V}_t$, meaning our codebook contains a matrix that a statistical feedback scheme uses, regardless of b_U.

4.2. Diagonal Codebook Design

Because each $\mathbf{D}_m$, $m = 1, \ldots, 2^{b_D}$, is characterized by only M_t positive diagonal entries, we can adopt vector quantization techniques such as LBG-based algorithms [2]. We first randomly generate a large number of channel realizations $\{\mathbf{H}(t_0)\}$. For each channel realization, we compute $\mathbf{\Phi}(\mathbf{H}(t_0), \mathbf{R}_t)$ and find the optimal power shaping matrix $\mathbf{\Lambda}_{\text{opt}}$ in (5). We use the vectors consisting of the diagonal entries of $\{\mathbf{\Lambda}_{\text{opt}}\}$ as a training set for a vector quantization algorithm. The resulting representative vectors constitute the diagonal entries of $\{\mathbf{D}_m\}_{m=1}^{2^{b_D}}$.

5. SIMULATION

We present the block error rate performance of precoded full-rate LD codes with $M_t = M_r = L = 4$ to demonstrate the benefits of the proposed algorithm. We use TAST codes $\mathcal{T}_{4,4,4}$ [3] for our LD codes. Symbols are taken from the BPSK constellation, and ML decoding is performed by the sphere decoder. In generating random spatially correlated channel realizations, we use the virtual channel model [5], $\mathbf{H} = \mathbf{V}_r\mathbf{H}_c\mathbf{V}_t^\dagger$, choosing the DFT matrix as $\mathbf{V}_r = \mathbf{V}_t$. The entries of $\mathbf{H}_c$, $[\mathbf{H}_c]_{p,q}$, are independent and distributed as $\mathcal{CN}(0, \sigma_{p,q}^2)$ with $\sigma_{p,q}^2 = 1$ for $q = 1$, 0.5 for $q = 2$, 0.2 for $q = 3$, and 0.1 for $q = 4$, regardless of p. For temporal correlation, we choose Doppler spread $f_D = 30$ Hz and delay $\tau = 2$ ms.

Fig. 3 shows that at 10^{-3} block error rate level, infinite-rate feedback yields 2.4dB gain over the no precoding case, i.e. $\mathbf{W} = \mathbf{I}$. Using 12 bits of feedback, $(b_1, b_2, b_3, b_4, b_D) = (6, 2, 0, 0, 4)$, we get 1.8dB gain over the no precoding case, while statistical feedback reaps only 0.9dB gain. Comparison of the '$b_U = \infty$ bits, $b_D = \infty$ bits' curve with the '$b_U = \infty$ bits, $b_D = 0$ bit' curve reveals the importance of adaptive power shaping. The curve labeled '$b_U = 8$ bits, $b_D = 4$ bits (iid $\{\mathbf{U}_n\}$)' is to illustrate that a unitary codebook that does not take spatial correlation into account, i.e. using 1 in place of $[\mathbf{\Lambda}_t]_{\ell,\ell}^{1/2}\,\forall\ell$ in (7), receives a penalty of about 0.4dB compared with the same size codebook that follows (7). The curve labeled 'Statistical Feedback ($\mathbf{V}_0 = \mathbf{I}$)' corresponds to choosing $\mathbf{I}$ as the common right singular matrix $\mathbf{V}_0$ of $\mathbf{W}_k$, instead of $\mathbf{V}_0$ given in Section 4.

6. CONCLUSION

We proposed a precoder codebook design method for LD codes in spatially and temporally correlated channels, breaking the task into the unitary codebook design part and the diagonal codebook design part. We have proposed a systematic method for generating a codebook of unitary matrices matched to spatial correlation which are hard to obtain numerically due to the huge number of optimization variables. Simulations of block error rates of different feedback schemes show that the limited feedback scheme using the proposed method can achieve a significant portion of the gain that an infinite-rate feedback scheme achieves.

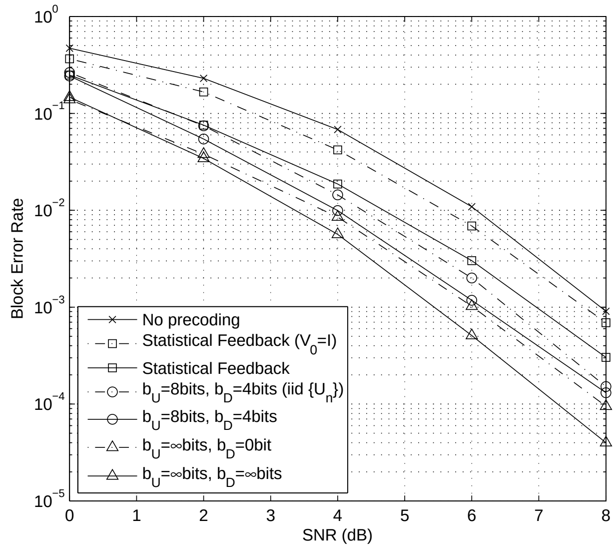

Fig. 3. Block error rate comparison of different feedback schemes

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