# Differential Beamspace MIMO for High-Dimensional Multiuser Communication

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***Abstract*—We develop a new multiple input multiple output (MIMO) transceiver architecture – Differential Beamspace MIMO (DB-MIMO) for high-dimensional MIMO systems. DB-MIMO enables linear quasi-coherent interference suppression between multiple spatially multiplexed data streams in conjunction with differential transmission. The differential nature of DB-MIMO is particularly attractive in emerging high-frequency systems, e.g. centimeter-wave (cmW) and millimeter-wave (mmW) systems, where the requirement of a phase-coherent local oscillator at the receiver can be challenging. The beamspace approach is naturally relevant to tame the complexity of massive MIMO due to the expected channel sparsity in beamspace, especially at cmW and mmW. We focus on downlink multiuser systems in which an access point equipped with a high-dimensional antenna array serves multiple single-antenna mobile stations (MSs). First, the concept of dominant multi-beam selection is discussed that enables transceivers whose complexity tracks the number of MSs. Then the recently introduced concept of differential MIMO is applied in which a quasi-coherent channel estimate is obtained from differential measurements. This enables the development of low-complexity quasi-coherent DB-MIMO transceivers. Numerical results are presented to demonstrate the minimal performance loss of the low-complexity DB-MIMO transceivers in comparison with their full dimensional coherent counterparts.**

***Index Terms*—Interference Suppression, Differential Signaling, Millimeter-wave, Centimeter-wave, Massive MIMO**

## I. Introduction

Emerging wireless systems operating at centimeter-wave (cmW) and millimeter-wave (mmW) frequencies offer an attractive opportunity for meeting the exploding capacity demands on wireless networks. In addition to the larger bandwidths, the small wavelengths enable high-dimensional multiple-input multiple-output (MIMO) operation by packing many more critically (half-wavelength) spaced antennas in a given aperture [1], [2]. The large number of MIMO degrees of freedom can be exploited for a number of critical capabilities, including: higher antenna/beamforming gain [1]–[4]; higher spatial multiplexing gain; and highly directional communication with narrow beams.

The extremely narrow beamwidths at cmW and mmW enable the key capability of dense spatial multiplexing [1], [2]: reuse of spectral resources across distinct beams. Coupled with the larger bandwidths, this promises unprecedented network throughput gains in multiuser-MIMO (MU-MIMO) systems equipped with high dimensional antenna arrays at the access point (AP). However there are significant challenges to realizing the potential of high-frequency, high-dimensional MU-MIMO systems. The requirement of a phase coherent local oscillator at the receiver can be difficult to achieve at such high-frequencies [5]. Additionally the high dimension of the spatial signal space results in prohibitively high transceiver complexity if conventional MIMO techniques are used [1]–[3].

In this paper we develop a new transceiver architecture - Differential Beamspace MIMO (DB-MIMO) - for high-dimensional MIMO systems that enable linear quasi-coherent interference suppression between multiple spatially multiplexed data streams together with differential signaling, removing the requirement phase coherent local oscillators at the receiver. Furthermore the expected channel sparsity in beamspace at cmW and mmW frequencies makes beamspace MIMO (B-MIMO) communication [1], [2], [6] - multiplexing data onto orthogonal spatial beams - the natural route for reducing complexity in high-dimensional MIMO systems.

We focus on downlink MU-MIMO systems in which an AP equipped with a high-dimensional antenna array serves multiple mobile stations (MS). First, the concept of dominant multi-beam selection [7], [8], which enables transceivers whose complexity tracks the number of MSs, is discussed. Then the recently introduced concept of differential MIMO (D-MIMO) [9] is applied to the low-dimensional system induced via multi-beam selection. This enables a quasi-coherent estimate of the channel matrix to be obtained from differential measurements. This, in turn, enables the development of low-complexity quasi-coherent DB-MIMO multiuser transceivers. Numerical results are presented that demonstrate the minimal performance loss incurred by the low-complexity DB-MIMO transceivers when compared to their coherent counterparts.

## II. System Model

We focus on an AP equipped with a multi-antenna array communicating with $K$ single-antenna MSs. We examine the more challenging scenario of downlink communication - the uplink problem is well-studied [10] and can be formulated easily along the lines discussed here. Let the AP be equipped with an $n$-dimensional antenna which we consider to be a critically-sampled uniform linear array (ULA) for simplicity. We note that this model also captures the performance of APs equipped with continuous aperture lens antennas that perform analog beamforming [1], [2]. The received signal at the $k$-th MS is given by

$$r_k = \boldsymbol{h}_k^H \boldsymbol{x} + \nu_k \qquad (1)$$

where $\boldsymbol{x} = [x_1, \ldots, x_n]^T$ is the $n \times 1$ transmitted signal, $\boldsymbol{h}_k$ is the $n \times 1$ channel vector, and $\nu_k \sim \mathcal{CN}(0, \sigma^2)$ is additive

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white Gaussian noise (AWGN). Stacking the signals for all MSs in a $K \times 1$ vector $\boldsymbol{r} = [r_1, \cdots, r_K]^T$ we get the antenna domain system equation

$$\boldsymbol{r} = \boldsymbol{H}^H \boldsymbol{x} + \boldsymbol{\nu} \ , \ \boldsymbol{H} = [\boldsymbol{h}_1, \cdots, \boldsymbol{h}_K] \tag{2}$$

where $\boldsymbol{H}$ is the $n \times K$ channel matrix that characterizes the system and $\boldsymbol{\nu} \sim \mathcal{CN}(0, \sigma^2 \boldsymbol{I})$. Our focus is on the design of the linear precoding matrix $\boldsymbol{G}$ for the transmitted signal, $\boldsymbol{x} = \boldsymbol{G}\boldsymbol{s} = \sum_{i=1}^{K} \boldsymbol{g}_i s_i$, where $\boldsymbol{s}$ is the $K \times 1$ vector of independent symbols for different MSs. The overall system equation becomes

$$\boldsymbol{r} = \boldsymbol{H}^H \boldsymbol{G}\boldsymbol{s} + \boldsymbol{\nu} \ , \ E[\|\boldsymbol{x}\|^2] = \mathrm{tr}(\boldsymbol{G}\boldsymbol{\Lambda}_s \boldsymbol{G}^H) \leq \rho \tag{3}$$

where the second equality represents the constraint on total transmit power, $\rho$, and $\boldsymbol{\Lambda}_s = E[\boldsymbol{s}\boldsymbol{s}^H]$ denotes the diagonal correlation matrix of $\boldsymbol{s}$.

At mmW and cmW the channel can be accurately modeled via $n \times 1$ array steering vectors

$$\boldsymbol{h}_k = \sum_{i=0}^{N_p} \beta_{k,i} \boldsymbol{a}_n(\theta_{k,i}) \ , \ \boldsymbol{a}_n(\theta) = [e^{-j2\pi\theta i}]_{i \in \mathcal{I}(n)} \tag{4}$$

where $\theta = 0.5 \sin\phi$ and $\mathcal{I}(n) = \{\ell - (n-1)/2 : \ell = 0, 1, \cdots n-1\}$ is a symmetric set of indices centered around 0. The $\{\theta_{k,i}\}$ denote the normalized path angles and $\{\beta_{k,i}\}$ represent the complex path losses associated with the different paths for the $k^{\text{th}}$ MS with the $i = 0$-th path representing the line of sight (LoS) path. In this paper, we focus on purely LoS channels with $\theta_{k,0} = \theta_k$, $|\beta_{k,0}| = 1$, and $\beta_{k,i} = 0$ for $i \neq 0$ for all MSs.

### A. Beamspace System Model

The beamspace MIMO system representation is obtained from (2) via fixed beamforming at the transmitter. The columns of the beamforming matrix, $\boldsymbol{U}_o$, are steering vectors corresponding to $n$ fixed spatial frequencies/angles with uniform spacing $\Delta\theta_o = \frac{1}{n}$ [1], [2], [6]:

$$\boldsymbol{U}_o = \frac{1}{\sqrt{n}} \left[\boldsymbol{a}_n \left(i\Delta\theta_o\right)\right]_{i \in \mathcal{I}(n)} \tag{5}$$

which represent $n$ orthogonal beams that cover the entire spatial horizon ($-\pi/2 \leq \phi \leq \pi/2$), and form a basis for the $n$-dimensional spatial signal space. In fact, $\boldsymbol{U}_o$ is a unitary discrete Fourier transform (DFT) matrix.

The beamspace system representation is obtained by choosing $\boldsymbol{G} = \boldsymbol{U}_o \boldsymbol{G}_b$ in (3)

$$\boldsymbol{r} = \boldsymbol{H}_b^H \boldsymbol{G}_b \boldsymbol{s}_b + \boldsymbol{\nu} \ , \ \boldsymbol{H}_b = \boldsymbol{U}_o^H \boldsymbol{H} = [\boldsymbol{h}_{b,1}, \cdots, \boldsymbol{h}_{b,K}] \tag{6}$$

where $\boldsymbol{s}_b = \boldsymbol{s}$ represents the beamspace symbol vector and $\boldsymbol{G}_b$ is the beamspace precoder. $\boldsymbol{x}_b = \boldsymbol{G}_b \boldsymbol{s}_b$ represents the precoded beamspace transmit signal vector. Since $\boldsymbol{U}_o$ is a unitary matrix, the beamspace channel matrix $\boldsymbol{H}_b$ is a completely equivalent representation of $\boldsymbol{H}$.

### B. Beam Selection

The most important property of $\boldsymbol{H}_b$ is that it has a sparse structure representing the directions of the different MSs, as illustrated in Fig. 1(a) for LoS links. The $k$-th column $\boldsymbol{h}_{b,k} = \boldsymbol{U}_o^H \boldsymbol{h}_k$ (the rows in Fig. 1(a)) is the beamspace representation of the $k$-th MS channel and has a few dominant entries near the true LoS direction $\theta_k$ of the MS. This sparse nature of the beamspace channel is exploited for designing reduced-complexity beamspace precoders that deliver near-optimal performance through the concept of beam selection.

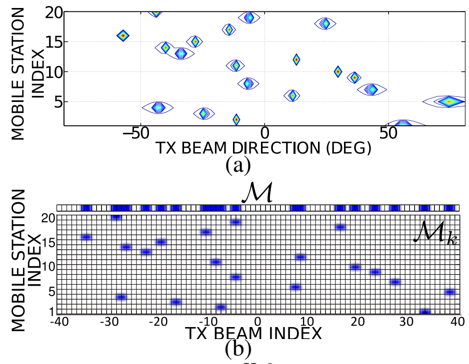


Fig. 1: (a) Contour plot of $|\boldsymbol{H}_b^H|^2$ for a ULA with $n = 81$, representing the *beamspace channel vectors* (rows) for 20 MSs randomly distributed between $\pm 90^o$ (b) Illustration of beamspace channel sparsity masks $\mathcal{M}_k$ and $\mathcal{M}$ for the $\boldsymbol{H}_b$ in (a).

We define the following sets of beam indices – *sparsity masks* – that represent the dominant beams that are selected for transmission at the AP (see Fig. 1(b)) [7], [8]:

$$\mathcal{M}_k = \left\{ i \in \mathcal{I}(n) : |h_{b,k}(i)|^2 \geq \gamma_k \max_i |h_{b,k}(i)|^2 \right\}$$
$$\mathcal{M} = \bigcup_{k=1,\cdots,K} \mathcal{M}_k \tag{7}$$

where $\mathcal{M}_k$ is the sparsity mask for the $k$-th MS, determined by the threshold $\gamma_k \in (0, 1)$. This beam selection is equivalent to selecting a subset of $p = |\mathcal{M}|$ rows of $\boldsymbol{H}_b$ resulting in the following low-dimensional system equation

$$\boldsymbol{r} = \tilde{\boldsymbol{H}}_b^H \tilde{\boldsymbol{G}}_b \boldsymbol{s}_b + \boldsymbol{w} \ , \ \tilde{\boldsymbol{H}}_b = [\boldsymbol{H}_b(\ell, :)]_{\ell \in \mathcal{M}} \ . \tag{8}$$

where $\tilde{\boldsymbol{H}}_b$ is the $p \times K$ beamspace channel matrix corresponding to the selected beams and $\tilde{\boldsymbol{G}}_b$ is the corresponding $p \times K$ precoder matrix, where $p \leq n$

For a given $\boldsymbol{H}$, the total multiuser channel power is defined as $\sigma_c^2 = \mathrm{tr}(\boldsymbol{H}\boldsymbol{H}^H) = \mathrm{tr}(\boldsymbol{H}_b \boldsymbol{H}_b^H)$, which under the simple LoS model is $\sigma_c^2 = n \sum_{k=1}^{K} |\beta_{k,0}|^2 = nK$. The thresholds $\{\gamma_k\}$ can be selected so that the $k$-th column of $\tilde{\boldsymbol{H}}_b$ captures a significant fraction $\eta_k$ of the power of $\boldsymbol{h}_{b,k}$ (e.g $\eta_k \geq 0.9$). This, in turn, implies that the fraction $\eta$ of the channel power captured by $\tilde{\boldsymbol{H}}_b$ is at least $\min_{k=1,\ldots,K} \eta_k$.

Conversely, the sparsity masks $\mathcal{M}_k$ can be chosen to select the $m$ dominant (strongest) beams for each MS, that is an $m$-beam mask. This implicitly defines the $\{\gamma_k\}$ as the ratio between the power of strongest and $m$-th strongest beams for each user. For the simple LoS channel model this corresponds to selecting the $m$ orthogonal beams closest to the true LoS direction of the MS $\theta_k$. In this paper we use a 2-beam mask for complexity reduction (see Fig. 1(b)).

## III. Multiuser Differential Beamspace MIMO

In this section we apply the concept of D-MIMO [9] to the high-dimensional B-MIMO system. We focus on the low-dimensional $p \times K$ system (8) induced via beam selection.

### A. Differential Beamspace MIMO System Model

In this section we develop a complex baseband system model for the multiuser DB-MIMO system in the downlink.

Define the two transmitted symbol vectors for the current symbol and previous symbol corresponding to $K$ differential symbols $\boldsymbol{\Delta\psi} = [\Delta\psi_1, \Delta\psi_2, \ldots, \Delta\psi_K]^T$:

$$\boldsymbol{s} = [s_1, s_2, \ldots, s_K] \ , \ \boldsymbol{s}_\tau = [s_{1\tau}, s_{2\tau}, \ldots, s_{K\tau}] \ . \quad (9)$$

The $\Delta\psi_k$ are drawn from a symmetric constellation (e.g. BPSK, QPSK) and differentially encoded as $s_k = e^{j\Delta\psi_k} s_{k\tau}$ [11]. From (8) the multiuser B-MIMO system equations for the current and previous symbol periods are

$$\boldsymbol{r} = \tilde{\boldsymbol{H}}_b^H \tilde{\boldsymbol{G}}_b \boldsymbol{s} + \boldsymbol{\nu} \ , \ \boldsymbol{r}_\tau = \tilde{\boldsymbol{H}}_b^H \tilde{\boldsymbol{G}}_b \boldsymbol{s}_\tau + \boldsymbol{\nu}_\tau \quad (10)$$

where $\boldsymbol{r}$ and $\boldsymbol{r}_\tau$ are the current and previous receive signal vectors respectively and $\boldsymbol{\nu}$ and $\boldsymbol{\nu}_\tau$ are independent. The multiuser DB-MIMO system equation is obtained as [9]

$$\begin{aligned}
\boldsymbol{z} &= \mathrm{vec}(\boldsymbol{r}\boldsymbol{r}_\tau^H) && (11)\\
&= \mathrm{vec}(\tilde{\boldsymbol{H}}_b^H \tilde{\boldsymbol{G}}_b \boldsymbol{s}\boldsymbol{s}_\tau^H \tilde{\boldsymbol{G}}_b^H \tilde{\boldsymbol{H}}_b^H) + \mathrm{vec}(\tilde{\boldsymbol{H}}_b^H \tilde{\boldsymbol{G}}_b \boldsymbol{s}\boldsymbol{\nu}_\tau^H) && (12)\\
&+ \mathrm{vec}(\boldsymbol{\nu}\boldsymbol{s}_\tau^H \tilde{\boldsymbol{G}}_b^H \tilde{\boldsymbol{H}}_b^H) + \mathrm{vec}(\boldsymbol{\nu}\boldsymbol{\nu}_\tau^H) && (13)\\
&= (\tilde{\boldsymbol{H}}_b^T \otimes \tilde{\boldsymbol{H}}_b^H)\mathrm{vec}(\tilde{\boldsymbol{G}}_b \boldsymbol{s}\boldsymbol{s}_\tau^H \tilde{\boldsymbol{G}}_b^H) + \boldsymbol{w}_1 + \boldsymbol{w}_2 + \boldsymbol{w}_3 && (14)\\
&= (\tilde{\boldsymbol{H}}_b^* \otimes \tilde{\boldsymbol{H}}_b)^H (\tilde{\boldsymbol{G}}_b^* \otimes \tilde{\boldsymbol{G}}_b)\mathrm{vec}(\boldsymbol{s}\boldsymbol{s}_\tau) + \boldsymbol{w} && (15)\\
&= \tilde{\boldsymbol{H}}_{b,d}^H \tilde{\boldsymbol{G}}_{b,d} \boldsymbol{\chi} + \boldsymbol{w} && (16)
\end{aligned}$$

where $\otimes$ denotes the Kronecker product [12], $\tilde{\boldsymbol{H}}_{b,d} = (\tilde{\boldsymbol{H}}_b^* \otimes \tilde{\boldsymbol{H}}_b)$ is the low-dimensional DB-MIMO channel matrix, $\tilde{\boldsymbol{G}}_{b,d} = (\tilde{\boldsymbol{G}}_b^* \otimes \tilde{\boldsymbol{G}}_b)$ is the low-dimensional DB-MIMO precoder, and $\boldsymbol{w}$ is the noise. We note that $\boldsymbol{w}$ is non-Gaussian and the second order statistics of $\boldsymbol{w}$ can be characterized in terms of $\tilde{\boldsymbol{H}}_b$, $\tilde{\boldsymbol{G}}_b$, and $\sigma^2$ as in [9]. The differential transmit symbol vector is $\boldsymbol{\chi} = \mathrm{vec}(\boldsymbol{s}\boldsymbol{s}_\tau)$ where the differential symbol for the $k$-th MS is encoded in the $k + K(k-1)$-th element of $\boldsymbol{\chi}$, $\chi_{k+K(k-1)} = s_k s_{k\tau}^*$.

**Note:** We assume that MSs operate independently, so only a subset of the full differential receive measurements $\boldsymbol{z}$ can be calculated. Specifically $z_{k+K(k-1)} = r_k r_{k\tau}^*$ is the element of $\boldsymbol{z}$ that can be calculated at the $k$-th MS and that corresponds to the $k+K(k-1)$-th element of $\boldsymbol{\chi}$ that contains the information symbol for the $k$-th MS. However, in general if the MSs are able to cooperate any arbitrary subset of $\boldsymbol{z}$ can be calculated.

### *B. Quasi-Coherent DB-MIMO Precoders*

We consider two designs for the linear MU-MIMO precoder $\tilde{\boldsymbol{G}}_b$: the matched filter (MF) and Wiener filter (WF). For the low-dimensional B-MIMO system (8) the linear MU-MIMO precoders are given by [13]–[15]:

$$\tilde{\boldsymbol{G}}_b = \alpha\boldsymbol{F} = \alpha[\boldsymbol{f}_1, \boldsymbol{f}_2, \cdots, \boldsymbol{f}_K], \ \alpha = \sqrt{\frac{\rho}{\mathrm{tr}(\boldsymbol{F}\boldsymbol{\Lambda}_s\boldsymbol{F}^H)}} \quad (17)$$

$$\boldsymbol{F}_{MF} = \tilde{\boldsymbol{H}}_b \quad (18)$$

$$\boldsymbol{F}_{WF} = (\tilde{\boldsymbol{H}}_b\tilde{\boldsymbol{H}}_b^H + \zeta\boldsymbol{I})^{-1}\tilde{\boldsymbol{H}}_b \ , \ \zeta = \frac{\mathrm{tr}(\boldsymbol{\Sigma}_\nu)}{\rho} = \frac{\sigma^2 K}{\rho} \quad (19)$$

However as we will show in the next section, what the AP can actually estimate for computing the precoder is $\tilde{\boldsymbol{H}}_{b,o}$:

$$\tilde{\boldsymbol{H}}_b = \tilde{\boldsymbol{H}}_{b,o}\boldsymbol{\Lambda}_\psi \ , \ \boldsymbol{\Lambda}_\psi = \mathrm{diag}(e^{j\psi_1}, e^{j\psi_2}, \ldots, e^{j\psi_K}) \quad (20)$$

and $\boldsymbol{\Lambda}_\psi$ is unknown. From the equations for the linear MU-MIMO precoders, it is clear that the precoder applied at the AP is $\tilde{\boldsymbol{G}}_{b,o} = \tilde{\boldsymbol{G}}_b\boldsymbol{\Lambda}_\psi^*$. Plugging this into the expression for the differential precoder yields

$$\begin{aligned}
\tilde{\boldsymbol{G}}_{b,d,o} &= (\tilde{\boldsymbol{G}}_{b,o}^* \otimes \tilde{\boldsymbol{G}}_{b,o}) = (\tilde{\boldsymbol{G}}_b^*\boldsymbol{\Lambda}_\psi \otimes \tilde{\boldsymbol{G}}_b\boldsymbol{\Lambda}_\psi^*) && (21)\\
&= (\tilde{\boldsymbol{G}}_b^* \otimes \tilde{\boldsymbol{G}}_b)(\boldsymbol{\Lambda}_\psi \otimes \boldsymbol{\Lambda}_\psi^*) = \tilde{\boldsymbol{G}}_{b,d}(\boldsymbol{\Lambda}_\psi \otimes \boldsymbol{\Lambda}_\psi^*). && (22)
\end{aligned}$$

**Note:** From (16) and (22) is clear that applying $\tilde{\boldsymbol{G}}_{b,o}$ rather than $\tilde{\boldsymbol{G}}_b$ is equivalent to applying the intended DB-MIMO precoder to a modified differential transmit symbol vector $\boldsymbol{\chi}_o = (\boldsymbol{\Lambda}_\psi \otimes \boldsymbol{\Lambda}_\psi^*)\boldsymbol{\chi}$. Examining the expression $(\boldsymbol{\Lambda}_\psi \otimes \boldsymbol{\Lambda}_\psi^*)$, it is clear that it is an all-phase diagonal matrix with ones in the diagonal elements corresponding to information carrying entries of $\boldsymbol{\chi}$. So using $\tilde{\boldsymbol{G}}_{b,o}$ rather than $\tilde{\boldsymbol{G}}_b$ does not affect the interference suppression between the data streams or the ability of the MSs to detect the differential symbols.

### *C. DB-MIMO Channel Estimation*

While we focus on communication in the downlink, we assume that the system is operating in a time division duplexed (TDD) mode where the quasi-coherent channel $\tilde{\boldsymbol{H}}_{b,o}$ can be estimated in the uplink. The uplink noise free $p \times 1$ current and previous low-dimensional receive vectors at the AP are

$$\tilde{\boldsymbol{r}}_b = \tilde{\boldsymbol{H}}_b\boldsymbol{x} \ , \ \tilde{\boldsymbol{r}}_{b,\tau} = \tilde{\boldsymbol{H}}_b\boldsymbol{x}_\tau \quad (23)$$

where $\boldsymbol{x}$ and $\boldsymbol{x}_\tau$ are the $K \times 1$ vectors of current and previous MS transmit signals. The uplink (noise-free) multiuser DB-MIMO equation is

$$\begin{aligned}
\boldsymbol{z} &= \mathrm{vec}(\tilde{\boldsymbol{r}}_b\tilde{\boldsymbol{r}}_{b,\tau}^H) = \mathrm{vec}(\tilde{\boldsymbol{H}}_b\boldsymbol{x}\boldsymbol{x}_\tau\tilde{\boldsymbol{H}}_b^H) && (24)\\
&= (\tilde{\boldsymbol{H}}_b^* \otimes \tilde{\boldsymbol{H}}_b)\mathrm{vec}(\boldsymbol{x}\boldsymbol{x}_\tau) = \boldsymbol{H}_{b,d}\boldsymbol{\chi} \ .
\end{aligned}$$

where $\boldsymbol{\chi} = \mathrm{vec}(\boldsymbol{x}\boldsymbol{x}_\tau)$ is the uplink differential transmit vector. From (25) it is clear that exciting each element of $\boldsymbol{\chi}$ allows $\boldsymbol{H}_{b,d}$ to be recovered from $\boldsymbol{z}$. Choosing $\boldsymbol{x} = \boldsymbol{e}_{M,k}$ and $\boldsymbol{x}_\tau = \boldsymbol{e}_{M,k'}$ where $\boldsymbol{e}_{M,i}$ is the $i$-th $M$-dimensional standard basis vector results in $\boldsymbol{\chi} = \boldsymbol{e}_{M^2,k+K(k'-1)}$. So sequentially exciting the $k$-th and $k'$-th MS allows the $k + K(k'-1)$-th column of the differential channel matrix $\tilde{\boldsymbol{H}}_{b,d}$ to be estimated. We note that this procedure requires cooperation across the MSs, and thus cannot be used if the MSs are assumed to be operating independently.

However, as shown in the previous section the linear precoder is calculated using the quasi-coherent estimate of the channel matrix $\tilde{\boldsymbol{H}}_{b,o}$. In contrast with estimating $\tilde{\boldsymbol{H}}_{b,d}$, estimating $\tilde{\boldsymbol{H}}_{b,o}$ does not require cooperation across the MSs. Considering the case where $\boldsymbol{x} = \boldsymbol{x}_\tau = \boldsymbol{e}_k$ the noise free differential receive signal is

$$\begin{aligned}
\boldsymbol{z} &= \mathrm{vec}(\tilde{\boldsymbol{h}}_{b,k}\tilde{\boldsymbol{h}}_{b,k}^H) = (\tilde{\boldsymbol{h}}_{b,k}^* \otimes \tilde{\boldsymbol{h}}_{b,k}) = && (25)\\
&\left[\tilde{h}_{b,k}(1)^*\tilde{\boldsymbol{h}}_{b,k}^T, \ \tilde{h}_{b,k}(2)^*\tilde{\boldsymbol{h}}_{b,k}^T, \ \ldots, \ \tilde{h}_{b,k}(p)^*\tilde{\boldsymbol{h}}_{b,K}^T\right]^T && (26)
\end{aligned}$$

where $\tilde{\boldsymbol{h}}_{b,k}$ is the $k$-th column of $\tilde{\boldsymbol{H}}_b$. Define the sub-vector of $\boldsymbol{z}$ obtained from the differential measurement $\boldsymbol{r}r_\tau^*(m_k)$:

$$\tilde{\boldsymbol{z}} = \tilde{\boldsymbol{r}}\tilde{r}_\tau^*(m_k) = \tilde{h}_{b,k}(m_k)^*\tilde{\boldsymbol{h}}_{b,k} \ . \quad (27)$$

It is clear that $\tilde{z}(m_k) = |\tilde{h}_{b,k}(m_k)|^2$ and that an estimate of $\tilde{\boldsymbol{h}}_{b,k}$ multiplied by an unknown phase is obtained as

$$\tilde{\boldsymbol{h}}_{b,k,o} = \frac{1}{\sqrt{|\tilde{z}(m_k)|}}\tilde{\boldsymbol{z}} = \tilde{\boldsymbol{h}}_{b,k}e^{-j\angle\tilde{h}_{b,k}(m_k)} \ . \quad (28)$$

In the noise free case the choice of $m_k$ is not significant so long as $\tilde{h}_{b,k}(m_k) \neq 0$, however when using noisy measurements $m_k$ will be chosen as $m_k = \arg\max_m |\tilde{h}_{b,k}(m)|^2$. Note that the information required for choosing $m_k$ can be obtained from the information used to calculate the beam selection masks $\mathcal{M}_k$ (see Sec. II-B). Performing this procedure for all $k = 1, \ldots, K$ MSs and arranging the vectors into a matrix yields the quasi-coherent channel estimate

$$\tilde{\boldsymbol{H}}_{b,o} = [\tilde{\boldsymbol{h}}_{b,1,o},\ \tilde{\boldsymbol{h}}_{b,2,o}, \ldots,\ \tilde{\boldsymbol{h}}_{b,K,o}] = \tilde{\boldsymbol{H}}_b \boldsymbol{\Lambda}_{\psi}^* \tag{29}$$

where $\boldsymbol{\Lambda} = \mathrm{diag}(e^{-j\angle \tilde{h}_{b,k}(m_1)}, \ldots, e^{-j\angle \tilde{h}_{b,k}(m_K)})$ is an unknown, all-phase diagonal matrix. As shown in Sec. III-B using $\tilde{\boldsymbol{H}}_{b,o}$ rather than $\tilde{\boldsymbol{H}}_b$ when calculating the precoders according to (17)-(19) does not impact the ability of the MSs to detect the differential symbols. Thus in the next section this method of obtaining a quasi-coherent estimate of the MU-MIMO channel matrix will be used when evaluating the performance of the quasi-coherent DB-MIMO precoders.

## IV. Numerical Results

This section presents numerical results that illustrate the performance of the proposed multiuser DB-MIMO transceiver architectures. The AP is equipped with a ULA of dimension $n = 81$ (linear 6" antenna at 80GHz) communicating with K=20 single-antenna MSs over LoS links.

Fig. 2 and 3 plot probability of error $P_e$ vs SNR for for the DB-MIMO precoders. The results are based on uncoded BPSK differential transmission with the $P_e$ calculated numerically over 1,00,000 symbol vectors. The channel realization (random MS locations) changes after every 200 symbols with the MSs located over the entire spatial horizon ($-0.5 \leq \theta_k \leq 0.5$). The curves in Fig. 2 were generated with the restriction that the MS LoS directions $\{\theta_k\}$ have minimum separation $\Delta\theta_{min} = \Delta\theta_o/4$ and the curves in Fig. 3 have minimum separation $\Delta\theta_{min} = \Delta\theta_o/2$. The $P_e$ performance of each DB-MIMO precoder is computed for two cases: one based on perfect knowledge of $\tilde{\boldsymbol{H}}_{b,o}$ and one based on estimated $\tilde{\boldsymbol{H}}_{b,o}$ with the same SNR as that for data communication. Additionally the ideal interference free coherent and DB-MIMO $P_e$ and the $P_e$ for the full-dimensional coherent precoders with and without channel estimation are included for comparison.

These results demonstrate that the use of the low-dimensional DB-MIMO precoders results in minimal performance loss when compared with the full-dimensional MIMO precoders. As shown in Fig. 3(b) when the minimum user separation is $\Delta\theta_{min} = \Delta\theta_o/2$ at $P_e = 10^{-3}$ there is only an SNR gap of about 1.5 dB between the full-dimensional coherent WF and the low-complexity DB-MIMO WF precoders for both perfect and estimated channels. This is only slightly larger than the theoretical SNR gap of 1.14 dB between BPSK and differential BPSK for single-input single-output systems at the same $P_e$ [11].

## V. Conclusions

We have presented a new transceiver architecture for high-dimensional MIMO systems - Differential Beamspace MIMO. By combining linear quasi-coherent spatial interference suppression in conjunction with differential transmission and B-MIMO communication, the DB-MIMO

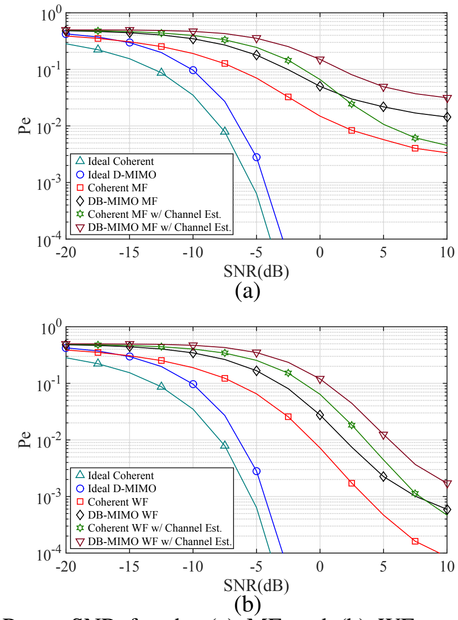


Fig. 2: Pe vs SNR for the (a) MF and (b) WF precoders with minimum MS separation $\Delta\theta_{min} = \frac{\Delta\theta_o}{4}$

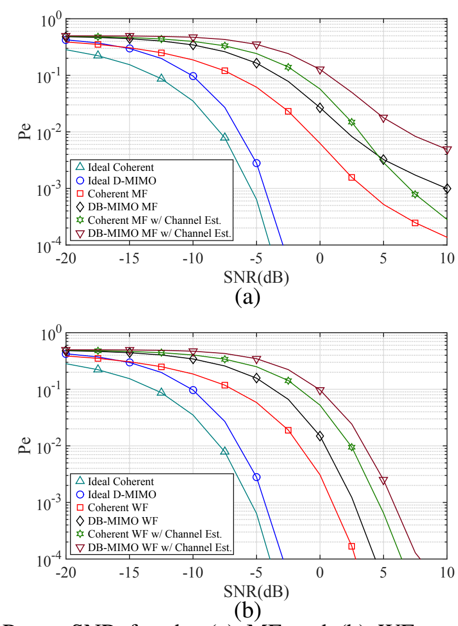


Fig. 3: Pe vs SNR for the (a) MF and (b) WF precoders with minimum MS separation $\Delta\theta_{min} = \frac{\Delta\theta_o}{2}$

transceivers are able significantly reduce the transceiver complexity of high-frequency, high-dimensional MU-MIMO systems. Additionally, we have shown how to obtain a quasi-coherent estimate of the B-MIMO channel matrix can be obtained from uplink differential measurements in TDD systems. Finally the numerical performance results demonstrate that using the low-complexity DB-MIMO transceivers incurs a mininimal performance loss when compared with their full dimensional coherent counterparts.

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