A Continuous Representation of Multi-Antenna Fading Channels and Implications for Capacity Scaling and Optimum Array Design

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***Abstract*— In this paper we introduce a continuous representation of multi-antenna (MIMO) fading channels that reveals the effect of array parameters and scattering characteristics on channel degrees of freedom and capacity. It is shown that the intrinsic degrees of freedom in the scattering environment provide an upper bound on the rank of the MIMO channel matrix (and hence the spatial multiplexing gain) and are determined by two key parameters: *angular spreads* and the *scattering correlation scales* seen by the transmitter and receiver. Similarly, the spatial signal spaces at the transmitter and receiver are determined by two key parameters each: the *aperture size* and the *smallest scale of signal variation*. For any given scattering environment, the continuous representation helps us determine the optimal number of antennas and the antenna spacings to maximize the spatial multiplexing gain (and hence capacity). In particular, we show that linear capacity scaling with the number of antennas is possible in ideally uncorrelated scattering environments with vanishing correlation scales. Conversely, for a non-vanishing scattering correlation scale, the capacity eventually saturates with the number of antennas.**

I. Introduction

Multiple Input Multiple Output (MIMO) systems employing arrays of antennas have emerged as a promising technology to dramatically increase the capacity and reliability of wireless communication links. Initial studies based on an idealized statistical channel model show a linear increase in MIMO channel capacity with the number of antennas [1], [2]. However, the idealized channel model is unrealistic and several models have been proposed to capture the physical propagation environment underlying realistic MIMO channels (see, e.g., [3], [4], [5]).

In this paper, we develop a continuous system representation to investigate the intrinsic degrees of freedom and capacity afforded by a given scattering environment. In the continuous representation, the transmitter and receiver are represented by continuous linear apertures in space (corresponding to linear arrays) and they interact with the scattering environment via an angular spreading function. The continuous representation is related to discrete MIMO channel models for uniform linear arrays (ULAs) via appropriate sampling of the transmit and receiver apertures.

The capacity of a MIMO channel is determined by the interaction between the spatial signal space and the scattering environment. The continuous representation clearly reveals this interaction and guides optimal ULA design (number of antennas and antenna spacing) to maximize the rank of the resulting MIMO channel matrix that determines the multiplexing gain of the MIMO system. It is well-known that in the limit of high signal-to-noise ratio (SNR) MIMO channel capacity scales linearly with the rank. Using the continuous representation we show that the rank of the MIMO matrix can be estimated using a few key parameters associated with the signal spaces and the scattering environment. The key signal space parameters are: the *array aperture* and the *smallest scale of signal variation* (that determines required antenna spacing). The corresponding key parameters of the scattering environment are: the *angular spread* and the *correlation scale* (which determines the angular extent of correlation in the scattering environment). In particular, for a given scattering environment, we provide a simple upper bound on the rank of all possible MIMO channel matrices that is solely a function of intrinsic scattering characteristics. We show that optimal array design corresponds to matching these signal space parameters to the scattering parameters. In particular, the optimal aperture equals the inverse of the scattering correlation scale and the optimal antenna spacing equals the inverse of the angular scattering spread. These insights apply both to the transmitter and receiver side. An important implication of these results is that if the scattering environment exhibits a non-vanishing correlation scale, the channel capacity eventually saturates as we increase the number of antennas. Indefinite linear capacity scaling is only possible in an ideally uncorrelated scattering environment corresponding to a vanishing correlation scale.

The next section introduces the continuous channel representation, and relates it to a discrete MIMO system with ULAs. In Section III we identify the key signal space and scattering parameters, provide a simple upper bound on the rank of a MIMO channel that is a function of the intrinsic characteristics of the scattering environment, and describe optimum array design. Section IV provides some numerical results to illustrate the capacity scaling behavior predicted by the continuous channel representation.

II. Continuous Channel Representation

In this section we develop a continuous representation for the spatial signals at the transmitter and the receiver and the scattering environment that connects them. This will be

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used to understand the interaction between the signals and the scattering environment.

We assume narrowband signaling and far-field scattering conditions. For simplicity, suppose that transmitter and receiver apertures are on a straight line in space (as in uniform linear arrays (ULAs)). Fig. 1 illustrates the continuous representation. The location on the transmitter aperture is denoted by γ_T and the direction of the outgoing plane wave is denoted by ϕ_T ($\phi_T = 0$ corresponds to the horizontal axis). We assume $-\pi/2 \leq \phi < \pi/2$ for half-plane propagation for simplicity. Define the normalized aperture and angle variables at the transmitter as

$$\nu_T = \gamma_T/\lambda, \quad \mu_T = \sin(\phi_T), \tag{1}$$

where λ is the wavelength of propagation. The corresponding variables at the receiver, ν_R and μ_R are defined similarly. The transmitted signal in the far-field, $X_T(\mu_T)$ at angle μ_T, is related to the signal on the aperture $x_T(\nu_T)$ by

$$X_T(\mu_T) = \int_{-\infty}^{\infty} x_T(\nu_T) e^{j2\pi\nu_T\mu_T} d\nu_T. \tag{2}$$

This is a well-known relation found in antenna textbooks (e.g., [6]) and is a Fourier transform relationship. Thus, $X_T(\mu_T)$ can be interpreted as the spatial spectrum of the signal $x_T(\nu_T)$ on the transmitter aperture. We will refer to $x_T(\nu_T)$ as the transmitted signal in the *aperture domain* and $X_T(\mu_T)$ the transmitted signal in the *scattering domain* as it directly interacts with the scattering environment. The effect of the scattering is introduced by the *spatial spreading function* $G_c(\mu_R, \mu_T)$ (see Fig. 1). It relates the far-field signals of the transmitter and receiver via

$$X_R(\mu_R) = \int_{-1}^{1} G_c(\mu_R, \mu_T) X_T(\mu_T) d\mu_T. \tag{3}$$

Note that the spreading function determines the coupling between the transmitted and received signals in the far-field (scattering domain). Furthermore, the integration limits in (3) correspond to half-plane propagation; that is, the scatterers are located on the horizons $-\pi/2 \leq \phi_T, \phi_R < \pi/2$ only. Hence the *maximum supports* of $G_c(\mu_R, \mu_T)$ in the scattering domains of the transmitter and receiver are $\mu_T, \mu_R \in [-1, 1)$ (see (1)).

A physical scattering environment corresponding to L propagation paths can be modelled as

$$G_c(\mu_R, \mu_T) = \sum_{l=1}^{L} \beta_l \delta(\mu_R - \mu_{R,l}) \delta(\mu_T - \mu_{T,l}). \tag{4}$$

where $\mu_{T,l}$ and $\mu_{R,l}$ denote the transmit and receive angles, respectively, and β_l the path gain associated with the l-th path.

At the receiver, the signal in the aperture domain, $x_R(\nu_R)$, is obtained from the far-field signal in the scattering domain, $X_R(\mu_R)$, via the inverse of (2) (due to reciprocity) as

$$x_R(\nu_R) = \int_{-1}^{1} X_R(\mu_R) e^{-j2\pi\nu_R\mu_R} d\mu_R. \tag{5}$$

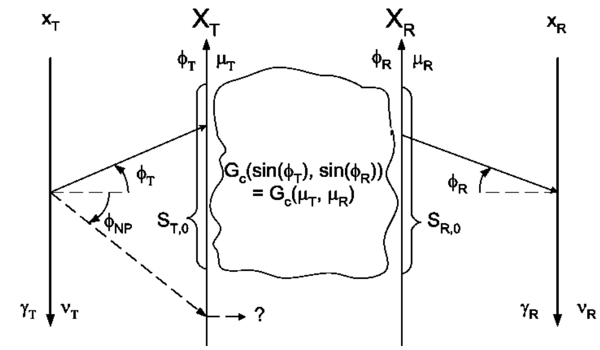

Fig. 1. Illustration of the continuous representation of signals in the aperture and scattering domains, and the scattering environment. Signals propagate through the channel if they couple to the scattering environment. For example, a signal transmitted in the direction ϕ_{NP} will not reach the receiver.

By combining (2), (3), and (5) we see that the transmitted and received signals are related by

$$x_R(\nu_R) = \int_{-\infty}^{\infty} H_c(\nu_R, \nu_T) x_T(\nu_T) d\nu_T, \tag{6}$$

$$H_c(\nu_R, \nu_T) = \int_{-1}^{1}\int_{-1}^{1} G_c(\mu_R, \mu_T) e^{-j2\pi\mu_R\nu_R} e^{j2\pi\mu_T\nu_T} d\mu_R \, d\mu_T. \tag{7}$$

Here (7) states that G_c and H_c are a Fourier transform pair as well. H_c represents the input-output spatial transfer function that characterizes the channel.

Practical MIMO systems correspond to a discrete representation of the channel in the aperture domains. In the usual notation

$$\mathbf{x}_R = \mathbf{H}\mathbf{x}_T, \tag{8}$$

$\mathbf{x}_T$ and $\mathbf{x}_R$ are the transmit and receive signal vectors corresponding to P transmit antennas and at Q receive antennas, respectively, and the $Q \times P$ channel matrix $\mathbf{H}$ describes the coupling between the transmit and receive signals. In relation to the continuous representation, $\mathbf{H}$ corresponds to a sampled version of the channel transfer function H_c. Assuming ULAs with antenna spacings $\Delta\nu_T$ and $\Delta\nu_R$ the (m, n)-th element of $\mathbf{H}$ is related to H_c as

$$H(m, n) = \frac{1}{\sqrt{PQ}} H_c(m\Delta\nu_R, n\Delta\nu_T), \tag{9}$$

where $0 \leq m \leq Q - 1$ and $0 \leq n \leq P - 1$. There exists a natural discrete representation of the channel in the scattering domain – the *virtual channel representation* introduced in [4].

III. Interaction Between the Spatial Signals and the Scattering Environment

In this section we investigate the *spatial signal spaces* at the transmitter and receiver and the *propagation space* defined by the scattering environment. We identify the key parameters that characterize the dimensions of the signal spaces and the degrees of freedom in the propagation space. The overall MIMO channel is determined by the interaction between the signal spaces and the propagation space. In particular, the degrees of freedom in the propagation space determine the inherent capacity and diversity afforded by the MIMO channel. Our development characterizes the interaction between the signal spaces and the propagation space and yields key insights into optimum array design (number of antennas and antenna spacing) for maximally exploiting the degrees of freedom in the propagation space.

A. Transmit and Receive Signal Spaces

Consider the transmit signal space. The dimensionality of the space of signals $x_T(\nu_T)$ in the aperture domain is determined by two key parameters: the *aperture size* A_T, and the *smallest scale of signal variation*, $\Delta\nu_T$, that can be interpreted as an interval on the aperture over which the signal is approximately constant. The smallest scale of variation implies that the aperture domain signals can be sampled without loss of information. In multi-antenna systems the antenna spacing corresponds to $\Delta\nu$ as in Section II. Due to the Fourier relation between $x_T(\nu_T)$ and $X_T(\mu_T)$, the aperture size A_T induces a smallest scale of variation in $X_T(\mu_T)$ in the scattering domain, $\Delta\mu_T = 1/A_R$, which corresponds to the *angular resolution* of the array. This is illustrated in Fig. 2. Similarly $\Delta\nu_T$ defines the angular bandwidth of $x_T(\nu_T)$ – the essential support of $X_T(\mu_T)$ – $S_T = 1/\Delta\nu_T$. The dimensionality of the transmit signal space is given by

$$N_T = A_T S_T + o(A_T S_T) \approx A_T S_T \qquad (10)$$

and it is determined by the signal support and smallest scale of variation in either the aperture or scattering domain

$$A_T S_T = \frac{A_T}{\Delta\nu_T} = \frac{S_T}{\Delta\mu_T}. \qquad (11)$$

The relation (10) for the dimensionality of the signal space is completely analogous to the well-known fact that the dimension of the space of signals of duration T and bandwidth W is $\mathcal{O}(TW)$ asymptotically [7]. The analogy is obtained by taking $T = A_T$ (or S_T) and $W = 1/\Delta\nu_T = S_T$ (or $1/\Delta\mu_T = A_T$).

Similarly, the dimensionality of the receive signal space is characterized by two key parameters: A_R and $\Delta\nu_R$ or, equivalently, $S_R = 1/\Delta\nu_R$ and $\Delta\mu_R = 1/A_R$. Then, $N_R = A_R S_R + o(A_R S_R) \approx A_R S_R$

B. Propagation Space

The propagation space depends on the scattering environment and is characterized by the angular spreading function $G_c(\mu_R, \nu_T)$ (see Fig. 1)). The degrees of freedom in the propagation space determine channel capacity and diversity and are characterized by two key parameters each on the transmitter and receiver side: angular spreads, $S_{T,o}$ and $S_{R,o}$, and the *correlation scales* in $G_c(\mu_R, \mu_T)$, $\Delta\mu_{T,o}$ and $\Delta\mu_{R,o}$. Note that the maximum value for $S_{T,o}$ and $S_{R,o}$ is 2. The correlation scales are analogous to smallest scales of variation and correspond to (angle) intervals over which $G_c(\mu_R, \mu_T)$ remains strongly correlated (and hence approximately constant). The degrees of freedom in the propagation space on the transmitter side are given by

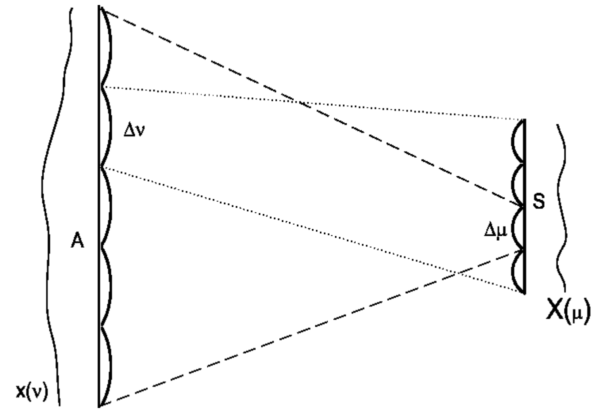

Fig. 2. Illustration of Fourier duality between the aperture domain (left) and the scattering domain (right). The signal support in one domain governs the smallest scale of variations in the signal in the dual domain, and vice versa. For example, the aperture size A corresponds to the smallest scale of variation in the scattering domain $\Delta\mu = 1/A$. Similarly, the smallest scale of variation in the aperture domain, $\Delta\nu$ corresponds to the signal support S in the scattering domain.

$$N_{T,o} = \frac{S_{T,o}}{\Delta\mu_{T,o}} + o\left(\frac{S_{T,o}}{\Delta\mu_{T,o}}\right) \qquad (12)$$

and correspond to the number of uncorrelated (independent in Rayleigh fading) scatterers that can be resolved at the transmitter side. Similarly, the degrees of freedom in the propagation space on the receiver side are given by

$$N_{R,o} = \frac{S_{R,o}}{\Delta\mu_{R,o}} + o\left(\frac{S_{R,o}}{\Delta\mu_{R,o}}\right) \qquad (13)$$

and correspond to the number of uncorrelated scatterers that can be resolved at the receiver side.

It is well-known that coherent capacity (when the receiver knows the channel) of a MIMO channel is proportional to the rank of the channel matrix $\mathbf{H}$. As we will see, the degrees of freedom in the propagation space determine the maximum achievable rank over all possible ULA configurations (antenna spacings and the number of antennas)

$$\text{rank}(\mathbf{H}) \leq \min(N_{T,o}, N_{R,o}). \qquad (14)$$

The richness of scattering (and diversity) is determined by the number of resolvable scatterers on the receiver side that are coupled with each resolvable scatterer on the transmitter, and vice versa [4]. We note that rich scattering (all scatterers on the transmit side coupling with all scatterers on the receive side) necessarily requires multiple intermediate scatterers (multiple bounces) between those seen by the transmitter and the receiver in the scattering domain.[1] Finally, we note that if $\Delta\mu_{T,o} = \Delta\mu_{R,o} = 0$, the propagation space has infinite dimensions and corresponds to an ideally uncorrelated scattering environment.

[1] Personal communication with Prof. Paulraj.

C. Optimum Array Design: Interaction Between the Signal and Propagation Spaces

For any given scattering environment and array configurations, the degrees of freedom in the MIMO channel are determined by the interaction between the (transmit and receive) signal spaces and the propagation space. We are now in a position to characterize this interaction and to determine optimum array design for a given scattering environment to maximally exploit the degrees of freedom in the propagation environment.

Consider a given scattering environment characterized by $G_c(\mu_R, \mu_T)$. The key to optimum array design is to match the signal supports and smallest scales of variation in the *scattering domain* to angular spreads ($S_{T,o}$ and $S_{R,o}$) and correlation scales ($\Delta\mu_{T,o}$ and $\Delta\mu_{R,o}$) of $G_c(\mu_R, \mu_T)$. By optimum array design we mean optimal choices of aperture sizes (A_T and A_R) and smallest scales of variation ($\Delta\nu_T$ and $\Delta\nu_R$ that determine antenna spacing) that maximally exploit the inherent degrees of freedom in the propagation space. Specifically, the optimal values of these key signal space parameters are matched to the key propagation space parameters in the scattering domain

$$\begin{aligned} S_{T,opt} &= 1/\Delta\nu_{T,opt} = S_{T,o} \\ \Delta\mu_{T,opt} &= 1/A_{T,opt} = \Delta\mu_{T,o} \end{aligned} \quad (15)$$

$$\begin{aligned} S_{R,opt} &= 1/\Delta\nu_{R,opt} = S_{R,o} \\ \Delta\mu_{R,opt} &= 1/A_{R,opt} = \Delta\mu_{R,o}. \end{aligned} \quad (16)$$

Note that with the above matching the corresponding optimal dimensions of the *signal spaces* at the transmitter and receiver are equal to the degrees of freedom in the *propagation space* at the transmitter and receiver, respectively

$$N_{T,opt} = \frac{A_{T,opt}}{\Delta\nu_{T,opt}} = \frac{S_{T,o}}{\Delta\mu_{T,o}} = N_{T,o} \quad (17)$$

$$N_{R,opt} = \frac{A_{R,opt}}{\Delta\nu_{R,opt}} = \frac{S_{R,o}}{\Delta\mu_{R,o}} = N_{R,o}. \quad (18)$$

A MIMO system corresponding to the above optimum array design will maximally exploit the scattering environment as evident from the fact that the rank of the corresponding channel matrix $\mathbf{H}$ will be $\min(N_{T,o}, N_{R,o})$ which is equal to the propagation space upper bound identified in (14).[2]

In general, for mismatched array design the degrees of freedom in the channel at the transmitter and receiver are given by $N_{T,ch} = \min(N_T, N_{T,o})$ and $N_{R,ch} = \min(N_R, N_{R,o})$ and $\mathrm{rank}(\mathbf{H}) = \min(N_{T,ch}, N_{R,ch}) \leq \min(N_{T,o}, N_{R,o})$. It is worthwhile to see how the mismatch between the signal and propagation spaces results in a loss in the channel degrees of freedom and rank. By considering the interaction between the signal space and the propagation space in the scattering domain, we have the following different cases for N_{ch} (at the transmitter or receiver)

$$\begin{aligned} N_{ch} &= \min(A/\Delta\nu, S_o/\Delta\mu_o) = \frac{\min(A, 1/\Delta\mu_o)}{\max(\Delta\nu, 1/S_o)} \\ &= \begin{cases} AS_o, & A < 1/\Delta\mu_o \ , \ \Delta\nu \leq 1/S_o \\ S_o/\Delta\mu_o, & A \geq 1/\Delta\mu_o \ , \ \Delta\nu \leq 1/S_o \\ A/\Delta\nu, & A < 1/\Delta\mu_o \ , \ \Delta\nu > 1/S_o \\ 1/(\Delta\mu_o\Delta\nu), & A \geq 1/\Delta\mu_o \ , \Delta\nu > 1/S_o. \end{cases} \end{aligned} \quad (19)$$

Note that $M = A/\Delta\nu$ denotes the number of antennas that would be needed to cover the aperture A with spacing $\Delta\nu$. The feasible ranges for A and $\Delta\nu$ to maximally exploit the degrees of freedom in the propagation space are $A \geq 1/\Delta\mu_o$ and $\Delta\nu \leq 1/S_o$ to capture the finest scale of variation and the angular spread of the scattering environment, respectively. However, equalities, corresponding to (17) and (18), result in maximum degrees of freedom with minimum number of antennas and thus correspond to optimal choices. Sub-optimum array design occurs in (19) due to limitations of the scattering environment in the second case, due to array limitations in the third case, and due to a combination of both in first and fourth cases. In particular, the second case illustrates the important fact that if the scattering environment exhibits non-vanishing correlation scale, the degrees of freedom and hence channel capacity will eventually saturate as we increase the number of antennas (by increasing A or reducing $\Delta\nu$). This is in contrast to well-known capacity scaling results based on idealized i.i.d. channels that exhibit indefinite linear scaling with the number of antennas. Such idealized models correspond to the ideally uncorrelated scattering in which $\Delta\mu = 0$. In such a situation, it follows from the first case in (19) that for a given feasible antenna spacing, the channel degrees of freedom (and hence rank/capacity) will increase linearly with the number of antennas indefinitely (due to increase in $A = M\Delta\nu$).

IV. Numerical Examples: Capacity Scaling

The numerical examples in this section illustrate capacity scaling with the number of antennas and also show the effect of scattering correlation scale on capacity scaling. For simplicity, we consider a symmetric situation in which the parameters of the array and the scattering environment are identical at the transmitter and receiver sides: $P = Q = M$, $\Delta\nu_R = \Delta\nu_T = \Delta\nu$ (antenna spacing), $S_{T,o} = S_{R,o} = S_o = 2$ (maximum angular spreads), and $\Delta\mu_{T,o} = \Delta\mu_{R,o} = \Delta\mu_o$.

Given knowledge of $\mathbf{H}$ at the receiver, the maximum mutual information ("capacity") corresponding to an i.i.d. Gaussian input is given by [1] by

$$\begin{aligned} C(\mathbf{H}) &= \log_2 \det\left(\mathbf{I}_Q + \frac{\rho}{P}\mathbf{H}\mathbf{H}^H\right) \\ &= \sum_{k=1}^{\mathrm{rank}(\mathbf{H})} \log_2\left(1 + \frac{\rho}{P}\lambda_k^2\right) \end{aligned} \quad (20)$$

where λ_k^2 are the eigenvalues of $\mathbf{H}\mathbf{H}^H$, and ρ is the total transmit power (SNR) and we set it to 20 dB to represent a high

[2] This argument implicitly assumes that the scattering environment is non-degenerate in the sense that the set of scatterers on the receiver side that couple with the set of *resolvable* scatterers on the transmitter side includes all the *resolvable* scatterers at the receiver side, and vice versa (e.g., the k-diagonal model in [4]). An example of a degenerate channel which does not satisfy this condition is the pin-hole channel [8] that results in a rank-1 channel matrix despite maximum degrees of freedom at the transmitter and receiver. Our framework can account for this case as well – details will be reported elsewhere.

SNR situation. The ergodic capacity is given by $C = E[C(\mathbf{H})]$ where the expectation is taken over the statistics of $\mathbf{H}$.

We consider two cases for increasing M. In the first case, the antenna spacing is chosen to be optimal $\Delta\nu = 1/S_o$ and the number of antennas M is increased. The antenna aperture will increase linearly with M, $A = M\Delta\nu$, and we expect a corresponding linear increase in $\text{rank}(\mathbf{H})$ (and hence capacity) until A reaches the optimal point $A_{opt} = 1/\Delta\mu_o$ (see (15) and (16)). This yields the optimal number of antennas $M_{opt} = A_{opt}/\Delta\nu = S_o/\Delta\mu_o$. Beyond this point, increase in M will not increase rank but only the receive SNR (values of λ_k^2 due to increased energy capture - array gain), resulting in a logarithmic increase in capacity. In the second case, we keep the aperture A constant. Increasing M will decrease the antenna spacing $\Delta\nu = A/M$. Again, as reflected in the third situation in (19), the channel rank and hence capacity will increase with M until $\Delta\nu$ reaches the optimal spacing $\Delta\nu_o = 1/S_o = 0.5$ to cover the entire angular spread. This yields the optimal number of antennas $M_{opt} = A/\Delta\nu_o = AS_o$. Increasing M beyond this value will not increase rank but only the received SNR due to energy gain, resulting in logarithmic increase in capacity.

The scattering environment is simulated using the discrete physical model in (4). Note that due to the delta functions in (4), the correlation scale is vanishing; that is, $\Delta\mu_o = 0$. To simulate non-zero correlation scales, the spreading function needs to be smooth. A simple way of introducing the required smoothness on the scale of $\Delta\mu_o$ is to replace the delta functions in (4) with appropriately scaled sinc functions

$$G_c(\mu_R, \mu_T) = \sum_{l=1}^{L} \beta_l \,\text{sinc}\left(\frac{\mu_R - \mu_{R,l}}{\Delta\mu_{R,o}}\right) \text{sinc}\left(\frac{\mu_T - \mu_{T,l}}{\Delta\mu_{T,o}}\right) \tag{21}$$

The scattering environment is simulated using $L = 2,500$ paths with angles $\phi_T = \sin^{-1}(\mu_T)$ and ϕ_R uniformly distributed over the angular spreads. The path gains are assumed to be zero-mean i.i.d. complex Gaussian random variables with variance σ^2. The variance is adjusted so that the corresponding channel power is $\sigma_H^2 = \text{E}[\text{trace}(\mathbf{H}\mathbf{H}^H)] = PQ$. The ergodic channel capacity is estimated by averaging $C(\mathbf{H})$, given in (20), over 1,000 independent channel realizations.

Fig. 3 shows the ergodic capacity scaling results for three different situations. The situations (A) and (B) correspond to the first case in which M is increased and antenna spacing is fixed at the optimal point. The correlation scale $\Delta\mu_o = 0$ in (A), representing ideal uncorrelated scattering, and as evident from Fig. 3, the capacity linearly scales with M indefinitely since the propagation space has infinite degrees of freedom. In (B), $\Delta\mu_o = 0.15$, and as expected the capacity scales linearly until $M_{opt} = S_o/\Delta\mu_o \approx 13$, beyond which logarithmic increase is observed. The situation (C) corresponds to the second case in which the aperture is fixed at $A = 4$ and M is increased. Again, as expected, we see linear increase in capacity until $\Delta\nu = A/M = 1/S_o = 0.5$ corresponding to $M_{opt} = 8$. Beyond this point we see logarithmic increase in capacity as expected.

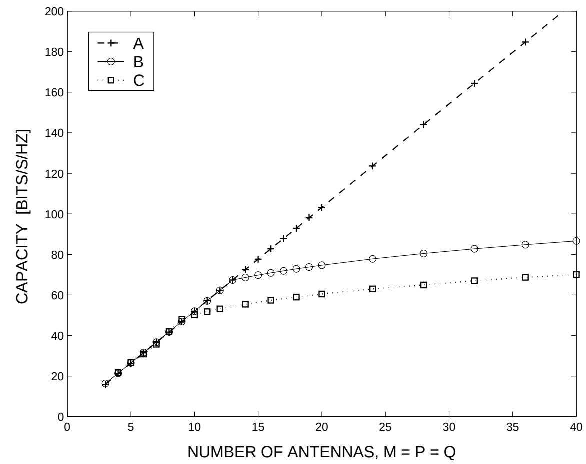

Fig. 3. Capacity scaling with the number of antennas $P = Q = M$. (A) and (B) correspond to the case when antenna spacing $\Delta\nu = 0.5$ is fixed and M is increased. (A) The scattering correlation scale $\Delta\mu_o = 0$ resulting in indefinite increase in capacity with M as expected. (B) The scattering correlation scale $\Delta\mu = 0.15$ resulting in linear capacity increase until $M_{opt} \approx 13$ followed by logarithmic increase as expected. (C) The aperture size is fixed, $A = 4$, and increase in M results in decreased antenna spacing. As expected, the capacity increases linearly until optimal antenna spacing $\Delta\nu = 0.5$ is achieved at $M_{opt} \approx 8$, followed by logarithmic increase.

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