Discrete Lens Array Optimization for MIMO Communication

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***Abstract*—Continuous Aperture Phased MIMO (CAP-MIMO) is a novel transceiver architecture that combines the concept of beamspace MIMO and analog beamforming to fully exploit the advantages of high-dimensional MIMO channels with the lowest possible transceiver complexity. CAP-MIMO theory dictates that the analog beamformer affect a spatial Fourier transform. A CAP-MIMO prototype uses a discrete lens array (DLA) designed for broadside focusing, which approximates the Fourier transform well near broadside but degrades at larger angles. This paper discusses the optimization of the DLA to achieve a closer approximation to the Fourier transform over a wider angular spread. Numerical results obtained using an iterative algorithm are presented that demonstrate the more uniform performance achieved by the optimized DLA.**

I. Introduction

Millimeter-wave communication systems, operating from 30-300 GHz [1], offer unique opportunities for meeting the increasing data rate demands on wireless communication systems. In addition to large bandwidths, the small-wavelengths allow for high-dimensional Multiple-Input Multiple-Output (MIMO) operation with relatively compact arrays. Continuous Aperture Phased MIMO (CAP-MIMO) is a novel transceiver architecture that combines the concept of beamspace MIMO - multiplexing data onto orthogonal spatial beams - and analog beamforming [2]. This allows CAP-MIMO to exploit the expected channel sparsity at mm-wave [1], [3] to achieve near-optimal performance with the lowest possible transceiver complexity [2], [4].

CAP-MIMO theory dictates that the ideal beamformer affect a spatial Fourier transform. However a prototype CAP-MIMO system makes use of a discrete lens array (DLA) designed for broadside focusing fed by an array of antennas for analog beamforming [2], [4]. This DLA design provides a good approximation to the Fourier transform near broadside that degrades at wider angles. However, DLAs that more closely approximate the ideal Fourier transform over a wider angular spread are desirable for channels with multipath or multiple users. This paper discusses the optimization of the DLA to achieve the best approximation of the ideal Fourier transform.

II. DLA Modeling

CAP-MIMO theory is based on a finite-dimensional system representation induced by critical sampling of the antenna apertures. Considering a linear antenna of length L, the critically sampled points are equivalent to a n-dimensional uniform linear array (ULA) of antennas, where $n = \left\lfloor \frac{2L}{\lambda} \right\rfloor$ is the maximum number of spatial modes supported by the antenna/ULA [2], [5], [6]. In this setting, the beamformer, which may be analog or digital, is represented by an $n \times n$ beamforming matrix $\mathbf{U}_b$ where each column of $\mathbf{U}_b$ represents an array excitation vector corresponding to a fixed beam.

The ideal beamforming matrix is the unitary discrete Fourier transform (DFT) matrix, $\mathbf{U}_b = \mathbf{U}_{dft}$. For critically spaced ULAs, a plane wave in the direction of angle ϕ corresponds to a spatial frequency, $\theta = 0.5\sin(\phi)$. The columns of $\mathbf{U}_{dft}$ are n orthogonal, unit-norm array steering/response vectors at fixed spatial frequencies with uniform spacing $\Delta\theta_o = \frac{1}{n}$ [5]:

$$\mathbf{U}_{dft} = \left[\mathbf{u}\left(\Delta\theta_o i\right)\right]_{i\in\mathcal{I}(n)}, \ \mathbf{u}(\theta) = \frac{1}{\sqrt{n}}\left[e^{-j2\pi\theta\ell}\right]_{\ell\in\mathcal{I}(n)} \quad (1)$$

where $\mathcal{I}(n) = \{i - (n-1)/2 : i = 0, \cdots, n-1\}$. The beamforming matrix for the DLA, $\mathbf{U}_b = \mathbf{U}_{dla}$, is determined by the DLA's aperture phase profile $\boldsymbol{\psi}$ and the focal surface geometry $(\mathbf{f}, \boldsymbol{\rho})$ of the n feed antennas, where (f_i, ρ_i) are the coordinates of the feed antenna intended to create a beam at $\phi = \sin^{-1}(2\Delta\theta_o i)$ (see Fig. 1a). $\mathbf{U}_{dla}$ can be decomposed as $\mathbf{U}_{dla} = \mathbf{P}(\boldsymbol{\psi})\mathbf{U}_{fa}(\mathbf{f}, \boldsymbol{\rho})$, where $\mathbf{P}$ is a diagonal matrix with $P_{ii} = e^{-j\psi_i}$ that models the aperture phase profile and $\mathbf{U}_{fa}$ models the propagation between the feed antennas and the DLA aperture. Let $\mathbf{d}_i$ denote the $n \times 1$ column vector of distances between the feed antenna with coordinates (f_i, ρ_i) and the DLA aperture critical sample points. Let $\mathbf{h}_i$ be the vector such that $\mathbf{h}_i \circ \mathbf{d}_i = \mathbf{1}$[1]. The columns of $\mathbf{U}_{fa}$ are

$$\mathbf{U}_{fa} = \left[\tilde{\mathbf{u}}(f_i, \rho_i)\right]_{i\in\mathcal{I}(n)}, \ \tilde{\mathbf{u}}(f_i, \rho_i) = c_i \mathbf{h}_i \circ e^{-j\frac{2\pi}{\lambda}\mathbf{d}_i} \quad (2)$$

where c_i is a scalar chosen to ensure $\tilde{\mathbf{u}}$ has unit norm. Thus the columns of $\mathbf{U}_{dla}$ are given by

$$\mathbf{U}_{dla} = \left[\hat{\mathbf{u}}(\boldsymbol{\psi}, f_i, \rho_i)\right]_{i\in\mathcal{I}(n)}, \ \hat{\mathbf{u}}(\boldsymbol{\psi}, f_i, \rho_i) = \mathbf{P}(\boldsymbol{\psi})\tilde{\mathbf{u}}(f_i, \rho_i) \ . \quad (3)$$

III. Optimization

To find the DLA that provides the best approximation to the DFT, we want to find the DLA phase profile $\boldsymbol{\psi}$ and focal surface geometry $(\mathbf{f}, \boldsymbol{\rho})$ that minimizes the objective function

$$\|\mathbf{U}_{dft} - \mathbf{U}_{dla}\|_F^2 = \sum_{i\in\mathcal{I}(n)} \|\mathbf{u}(\Delta\theta_o i) - \hat{\mathbf{u}}(\boldsymbol{\psi}, f_i, \rho_i)\|_2^2 \ , \quad (4)$$

[1] $\circ$ is the Hadamard, or entrywise, product and $\mathbf{1}$ is the all ones vector.

which is equivalent to maximizing

$$\mathrm{Re}\left[\mathrm{tr}(\mathbf{U}_{\mathrm{dft}}^{\mathrm{H}}\mathbf{U}_{\mathrm{dla}})\right] \ . \tag{5}$$

Since the columns of $\mathbf{U}_{dla}$ and $\mathbf{U}_{dft}$ correspond to array excitation vectors, each is defined in terms of some arbitrary phase origin. While the choice of phase origin does not affect the beam produced by each column, it does impact the value of the objective functions (4) and (5). Thus must we replace $\mathbf{U}_{dla}$ with $\tilde{\mathbf{U}}_{dla} = \mathbf{U}_{dla}\mathbf{B}$, where $\mathbf{B}$ is a diagonal matrix of all-phase complex numbers. $\mathbf{B}$ is found by minimizing (4) over the phases of the diagonal elements $B_{ii} = \exp(j\gamma_i)$ for a fixed $\boldsymbol{\psi}$, $\mathbf{f}$, and $\boldsymbol{\rho}$, resulting in $\gamma_i = -\angle\left[\mathbf{u}\left(\Delta\theta_o i\right)^H \hat{\mathbf{u}}(\boldsymbol{\psi}, f_i, \rho_i)\right]$.

To perform the optimization we adopt an iterative algorithm. At the k^{th} iterate $\boldsymbol{\psi}^{k+1}$, $(\mathbf{f}^{k+1}, \boldsymbol{\rho}^{k+1})$, and $\mathbf{B}^{k+1}$ are calculated in two steps. First, $\boldsymbol{\psi}^{k+1}$ is calculated by optimizing with $(\mathbf{f}, \boldsymbol{\rho}) = (\mathbf{f}^k, \boldsymbol{\rho}^k)$ and $\mathbf{B} = \mathbf{B}^k$ fixed. Writing (5) as $\mathrm{Re}\left[\mathrm{tr}(\mathbf{MP})\right] = \sum_{i=1}^{n} \mathrm{Re}\left[M_{ii}P_{ii}\right]$, where $\mathbf{M} = \mathbf{U}_{fa}\mathbf{B}\mathbf{U}_{dft}^H$, it is clear from $P_{ii} = \exp(-j\psi_i)$ that the optimal $\boldsymbol{\psi}$ is $\psi_i = \angle M_{ii}$. Second, $(\mathbf{f}^{k+1}, \boldsymbol{\rho}^{k+1})$ and $\mathbf{B}^{k+1}$ are updated by optimizing with $\boldsymbol{\psi} = \boldsymbol{\psi}^{k+1}$ fixed. While there is no closed form solution, fixing $\boldsymbol{\psi}$ does allow (4) to be minimized by individually minimizing $\|\mathbf{u}\left(\Delta\theta_o i\right) - \hat{\mathbf{u}}(\boldsymbol{\psi}, f_i, \rho_i)B_{ii}\|_2^2$ for each i. This is achieved by searching for the minimum over a specified set of possible f_i and ρ_i. At each point in the searching set, B_{ii} is calculated for the fixed $\boldsymbol{\psi}$, f_i, and ρ_i. The algorithm initializes the DLA to $\boldsymbol{\psi}^0$, $(\mathbf{f}^0, \boldsymbol{\rho}^0)$, and B^0 for a broadside focusing DLA with focal distance F_{init} [4]. Then after each iterate the value of (4) for $\tilde{\mathbf{U}}_{dla}(\boldsymbol{\psi}^{k+1}, \mathbf{f}^{k+1}, \boldsymbol{\rho}^{k+1})$ is compared to the value for $\tilde{\mathbf{U}}_{dla}(\boldsymbol{\psi}^{k}, \mathbf{f}^{k}, \boldsymbol{\rho}^{k})$. If it has decreased the algorithm continues iterating, otherwise it returns $\boldsymbol{\psi}^{k+1}$ and $(\mathbf{f}^{k+1}, \boldsymbol{\rho}^{k+1})$ as the solution.

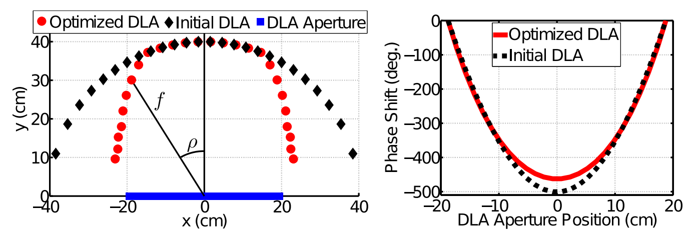
IV. Numerical Results

Using this algorithm, we have generated the following results for a one dimensional DLA of length $L = 40\text{cm}$ operating at $f_c = 10\text{GHz}$, which results in $n = 26$. The f_i search values were from 25cm to 40cm in 5mm increments and the ρ_i search values correspond to 51 points uniformly spaced in spatial frequency about $-\Delta\theta_o i$.

The value of (4) is 5.94 for the optimized DLA compared to 15.93 for a broadside focusing DLA with $F = 40\text{cm}$ ($\|\mathbf{U}_{dft}\|_F^2 = \|\mathbf{U}_{dla}\|_F^2 = n = 26$). Fig. 1 shows the focal surfaces and phase profiles of the initial and optimized DLAs. It can be seen that most significant change between the initial and optimized DLAs occurs in the focal surface geometry. Fig. 2 plots the diagonal values of $|\mathbf{U}_{dft}^H\mathbf{U}_b|^2$ for the ideal DFT and the optimized and initial DLAs. Physically, the diagonal values of $|\mathbf{U}_{dft}^H\mathbf{U}_b|^2$ represent how much power each beam (feed antenna) couples to a far field receiver at the intended spatial angle. It is clear that the optimized DLA obtains a much better approximation to the ideal DFT than the broadside focusing DLA. For instance the optimized DLA stays within 50% (-3dB) of the ideal DFT's power over an angular range greater than $\pm 60^\circ$ compared to $\pm 35^\circ$ for the initial DLA.

V. Conclusions

We have presented an optimization problem for obtaining DLA designs that provide a better approximation to the ideal DFT beamformer. The numerical results, obtained using our iterative algorithm, demonstrate that the optimized DLA provides a more uniform performance over a wider angular spread than the broadside focusing DLA. While our main motivation has been designing DLAs for CAP-MIMO, the results may be applied to any application where a DLA with uniform performance over a wide angular spread is desirable. These results open several avenues for further research. Extension to planar apertures will be key in applying these results to actual DLA prototypes. Although the iterative algorithm converges to the same solution regardless of F_{init}, suggesting that it is optimal, this is not necessarily the true. Thus joint optimization over $\boldsymbol{\psi}$ and $(\mathbf{f}, \boldsymbol{\rho})$ should be explored.



(a) (b)

Fig. 1: The focal surface (a) and phase profile (b) of the initial broadside focusing and optimized DLAs

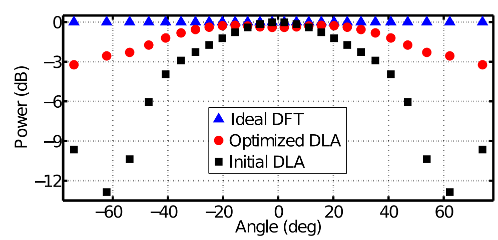


Fig. 2: Diagonal values of $|\mathbf{U}_{dft}^H\mathbf{U}_b|^2$

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