High Frequency Differential MIMO: Basic Theory and Transceiver Architectures

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Abstract—We propose a new multiple input multiple output (MIMO) transceiver architecture – Differential MIMO (D-MIMO) – that enables linear interference suppression between multiple spatially multiplexed and differentially encoded data streams. The D-MIMO transceiver architecture is particularly attractive in emerging high-frequency systems, such as millimeter-wave systems, in which the requirement of a phase-coherent local oscillator at the receiver can be challenging. A direct application of conventional linear interference suppression techniques is not possible with differential communication. Thus, we first develop a general model for D-MIMO systems, with a corresponding D-MIMO channel matrix, that forms the basis of the development in this paper. A surprising result is that a quasi-coherent version of the underlying channel matrix can also be estimated from the D-MIMO matrix, making conventional linear interference suppression possible as well. This leads to two D-MIMO transceiver architectures that are developed. Numerical results illustrate the promising and nearly identical performance of the proposed transceivers and the communication breakdown that can occur without interference suppression.

Index Terms—Spatial Multiplexing, Interference Suppression, Differential Signaling, Millimeter-wave, Kronecker Product

I. INTRODUCTION

There is growing interest in exploring higher frequencies (>5GHz) for meeting the Gigabit data rates and operational requirements of emerging wireless technologies. In particular, millimeter-wave (mmW) communication systems, ranging from 30GHz-300GHz, are emerging as a promising technology for 5G wireless [1]. In addition to the orders-of-magnitude larger bandwidth available at such high frequencies compared to existing systems, the small wavelengths make high-dimensional MIMO operation very attractive as well. Furthermore, the highly directional and quasi-optical nature of propagation at such high frequencies makes beamspace MIMO techniques and architectures naturally relevant [2]–[4]. However, many technical challenges need to be addressed before the full potential of mmW MIMO can be realized.

One challenging issue at mmW and high frequencies is phase-coherence between the transmitter and the receiver and the associated phase noise [5]. In single channel systems, an attractive solution is differential communication [6]. However, the use of differential communication is challenging in a MIMO system due to the interference between different spatial data streams. Differential space-time block coding schemes (e.g. [7]–[10]) do not support multiple spatial data streams. On the other hand, the differential spatial multiplexing scheme in [11] performs linear interference suppression based on the receive correlation matrix of the coherent MIMO channel. Finally, differential space-time coding that achieves the full rate and does not require knowledge of the coherent MIMO channel (e.g. [12], [13]) require complex non-linear detectors.

Linear interference suppression techniques that have been extensively studied for coherent MIMO systems, require knowledge of a coherent estimate of the MIMO channel matrix, and thus cannot be directly used.

In this paper, we propose new differential MIMO transceiver architectures that enable linear MIMO interference suppression within the context of differential communication. We first develop a general model for D-MIMO systems and identify a fundamental system equation, with a corresponding D-MIMO channel matrix, that forms the basis of the development in this paper. A surprising result is that a quasi-coherent version of the underlying channel matrix can also be estimated from the D-MIMO matrix, making conventional linear interference suppression feasible as well. This leads to the development of two D-MIMO transceiver architectures. Numerical results illustrate the promising and near-identical performance of the proposed transceivers compared to idealized systems in which there is no interference, and the communication breakdown that can occur without interference suppression. The results in this paper are based on a sub-system of the general model that suggests new avenues for future research.

II. DIFFERENTIAL SIGNALING AND RECEIPTION

Differential communication is typically used when a phase-coherent local oscillator is not available at the receiver, resulting in an unknown phase offset between the transmitter and receiver, and possibly even a sufficiently small frequency offset [6]. This problem is even more acute at high frequencies, such as mmW [5]. Consider a constant modulus constellation in which the transmitted symbols are of the form \( s = Ae^{j\phi} \) for some given fixed \( A \). Let \( A = 1 \) for simplicity. In a differential communication system, information is typically encoded in the phase difference \( \Delta \phi \) between the current transmit symbol \( s = s(t) \) and previous transmit symbol \( s_\tau = s(t - T) \) where \( T \) is the symbol period; that is,

\[
s = Ae^{j\phi} = e^{j\Delta \phi} s_\tau ; \quad s_\tau = Ae^{j\phi_\tau}.
\]

We assume that the differential symbols \( \Delta \phi \) are chosen randomly from a symmetric constellation, such as QPSK, and are independent across time. It follows that \( e^{j\Delta \phi} \) is zero mean and independent of \( s_\tau \). Under these assumptions, the following
can be readily shown:

\[
E[s_r] = 0 ; \quad E[s] = E[e^{j\phi}]E[s_r] = 0
\]

\[
|s|^2 = |s_r|^2 = A^2 = 1
\]

\[
ss^*_r = e^{j\Delta \phi}|s_r|^2 ; \quad E[ss^*_r] = 0 \quad (2)
\]

which also specifies the second-order statistics of the entire sequence of symbols, under the assumption that the starting symbol, \(s_0\), at time zero satisfies \(E[s_0] = 0\) and \(E[|s_0|^2] = A^2 = 1\), which can readily satisfied. The received signals and the differential measurements are

\[
r = e^{j\phi_0}s + v ; \quad r_r = e^{j\phi_c}s_r + v_r \quad \quad (3)
\]

\[
r_r^s = ss^*_r + sv^*_r + vs^*_r + vv^*_r = e^{j\Delta \phi} + w , \quad (4)
\]

where \(v, v_r, \) and \(w = sv^*_r + vs^*_r + vv^*_r\) represent noise. A key assumption is that the unknown phase offset \(\phi_c\), remains constant (or varies sufficiently slowly) over consecutive symbols, thereby enabling the detection of the differentially encoded symbols \(\Delta \phi\) from \(r_r^s\) in (4).

### III. Differential MIMO System Model

In this section, we develop a complex baseband model for the differential MIMO system. Consider a general \(n \times n\) MIMO system with \(n\) transmit and receive antennas. Define the two transmitted signal vectors for the current symbol and the previous symbol corresponding to \(n\) differential symbols

\[
\Delta \phi = [\Delta \phi_1, \Delta \phi_2, \ldots, \Delta \phi_n]^T ;
\]

\[
s = [s_1, s_2, \ldots, s_n]^T \quad (5)
\]

\[
s(r) = [s_1(t), s_2(t), \ldots, s_n(t)]^T \quad (6)
\]

\[
s_r = [s_1(t-T), s_2(t-T), \ldots, s_n(t-T)]^T \quad (8)
\]

\[
ss^*_r = [s_1s_1^*, s_2s_2^*, \ldots, s_ns_n^*]^T \quad (7)
\]

The corresponding received signals \(r, r_r\) are defined similarly. Finally, define the composite \(2n \times 1\) transmitted and received signal vectors as

\[
s_c = \begin{bmatrix} s \\ s_r \end{bmatrix} ; \quad r_c = \begin{bmatrix} r \\ r_r \end{bmatrix} \quad (10)
\]

The overall MIMO system equation for the two symbol vectors and the composite vector is

\[
r = Hs \quad r_r = H_r s_r \quad \quad r_c = H_c s_c \quad (11)
\]

where \(H = H(t)\) and \(H_r = H(t-T)\) and the \(2n \times 2n\) composite channel matrix \(H_c\) is given by

\[
H_c = \begin{bmatrix} H & 0 \\ 0 & H_r \end{bmatrix} \quad (12)
\]

A key assumption for differential communication is that \(H = H_r\); that is, the channel does not change across two symbol durations.

#### A. A Fundamental Equation

A key observation is that the following differential measurements are possible at the receiver

\[
R_c = r_c r_c^H = \begin{bmatrix} rr^H & r_r^H \\ r_r^H & r_r^H_r \end{bmatrix} \quad (13)
\]

Using (11) the system equation for these differential measurements at the receiver (without noise) is

\[
R_c = r_c r_c^H = H_c s_c r_r^H \quad H_c = H.Q_c H_c^H \quad (14)
\]

where \(Q_c = s_c r_r^H\) is of the same form as (13) for \(r, r_r^H\) and represents the possibilities for differential transmission. Using (12) and expanding out (14) we get

\[
R_c = \begin{bmatrix} rr^H & r_r^H \\ r_r^H_r & r_r^H_r \end{bmatrix} = \begin{bmatrix} H_s s_r^H H_r^H & H_s r_r^H \\ H_r s_c^H H_r^H & H_r s_c^H r_r^H \end{bmatrix} \quad (15)
\]

The matrix relation (15) represents a fundamental set of equations for understanding MIMO communication and interference suppression under differential signaling. Another version is obtained by vectorizing (14)

\[
z_c = \text{vec}(R_c) = [H_c^\tau \otimes H_c]x_c , \quad x_c = \text{vec}(Q_c) \quad (16)
\]

where we have used the relation

\[
\text{vec}(ADB) = [B^T \otimes A]\text{vec}(D) \quad (17)
\]

where \(\otimes\) denotes the Kronecker product [14]. An important special case of (17) for vectors \(a, b\) is

\[
\text{vec}(ab^H) = [b^\tau \otimes a]\text{vec}(I) = b^\tau \otimes a \quad (18)
\]

which will be used later in the paper.

#### B. An Important Sub-System

We consider an important sub-system of (15) and (16) that forms the basis of the investigation in this paper:

\[
in = H s \quad H_r s_r^H \quad (19)
\]

where we have used the assumption that \(H = H_r\). Vectorizing (19) we get

\[
z = H_d x ; \quad H_d = [H_c^\tau \otimes H] \quad (20)
\]

\[
z = \text{vec}(r_r^H) ; \quad x = \text{vec}(ss_r^H) \quad (21)
\]

\[
H_d = H_c^\tau \otimes H = H^* \otimes H \quad (25)
\]

\[
\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \otimes \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \quad (26)
\]
where $\Sigma_w = E[ww^H]$ is the covariance matrix of $w$, and $H_d H_d^H = (H_r^T H_r^T \otimes H H^H)$. The differentially encoded transmitted symbols in $x$ can then be estimated at the receiver by simply applying differential detectors, corresponding to the differential transmission scheme used, to the appropriate elements of $x_{est}$; see Remark 2 for the $n = 2$ case.

A. Signal and Noise Statistics

We now characterize the second-order statistics of $x$ and $w$ in (31). We consider zero-mean signal constellations for the differential symbols, with different differential symbols independent across time and data streams. This results in the following second-order statistics for $s$:

$$E[s] = E[ss^H] = 0, \quad E[ss^H] = 0$$

and

$$E[ss^H] = E[sr^H] = I_n$$

which in turn results in the following second-order statistics for $x = \text{vec}(ss^H)$:

$$E[x] = E[\text{vec}(ss^H)] = \text{vec}(E[ss^H]) = 0$$

and

$$E[xx^H] = E[\text{vec}(ss^H)\text{vec}(ss^H)]$$

$$= E[(s_r^T s_r) (s_r^T s_r^H)] = E[s_r^T s_r^H] = I_n$$

and

$$E[sr^H] = E[s_r^H] = I_n$$

Proposition 1 Assuming that the signal and noise are independent, and using the assumptions on the statistics of $v$ and $v_r$, it can be shown that

$$E[w] = 0$$

$$\Sigma_w = E[ww^H]$$

$$= \rho^2 (I_n \otimes H H^H) + \rho^2 (H_r^T H_r^T \otimes I_n)$$

$$+ \sigma^4 I_{n^2}$$

where the three terms in $\Sigma_w$ in (39) represent the covariance matrices of the corresponding terms in (32).

The noise statistics follow from the following calculations on the joint statistics of $w_1$, $w_2$, and $w_3$ in (32). Using (18), we first note that

$$w_1 = \sqrt{\rho} \text{vec}(H s v_r^H) = \sqrt{\rho}(v_r^* \otimes H s)$$

$$w_2 = \sqrt{\rho} \text{vec}(v_r^H H s) = \sqrt{\rho}(H_r^T s_r^* \otimes v)$$

$$w_3 = \text{vec}(v v_r^H) = (v_r^* \otimes v)$$

Now, the second-order statistics of $\{w_i\}$ are

$$E[w_1] = \sqrt{\rho} E[v_r^* H s] = 0$$

and

$$E[w_1 w_1^H] = \rho E[(v_r^* \otimes H s)(v_r^* \otimes H s)]$$

$$= \rho E[v_r^* v_r^* H H^H]$$

$$= \rho^2 E[v_r^* v_r^* H H^H]$$

We design $F_o$ using the minimum mean squared error (MMSE) criterion, assuming knowledge of the D-MIMO channel matrix $H_d$.

$$F_o = \arg\min_F \mathbb{E}[||x_{est} - x||^2]$$

$$= H_d^H \left(\rho^2 H_d H_d^H + \Sigma_w\right)^{-1} \tag{34}$$
Similarly, we have
\[ E[w_2] = E[w_3] = 0 \]  \hspace{1cm} (45)
\[ E[w_2^H w_2^H] = \rho \sigma^2 (H_r^r H_r^H \otimes I_n) \]  \hspace{1cm} (46)
\[ E[w_3^H w_3^H] = \sigma^2 I_n \otimes \sigma^2 I_n = \sigma^4 I_{n^2} \]  \hspace{1cm} (47)
Finally, it can be similarly shown that
\[ E[w_1^H w_1^H] = E[w_1^H w_1^H] = E[w_2^H w_2^H] = 0 \]  \hspace{1cm} (48)
Combining the above calculations leads to the second-order statistics of \( w \) given in Prop. 1.

Proposition 2 If \( HH^H \) has the eigenvalue decomposition \( HH^H = U \Lambda U^H \) and \( H_r H_r^H \) has the eigenvalue decomposition \( H_r H_r^H = U_r \Lambda_r U_r^H \), then the noise covariance matrix \( \Sigma_w \) admits the eigenvalue decomposition
\[ \Sigma_w = (U_r^r \otimes U) \Lambda (U_r^r \otimes U)^H \]  \hspace{1cm} (49)
\[ \Lambda = \rho \sigma^2 (\Lambda_r \otimes I_r) + \sigma^4 I_{n^2} \]  \hspace{1cm} (50)
where \( A \oplus B = (I \otimes A) + (B \otimes I) \) is the Kronecker sum [15].

This follows from Theorem 13.16 in [15] and the fact that \( U \) and \( U_r \) are unitary, and thus \( U_r^r \otimes U \) is also unitary. This result may be useful in analyzing the structure of \( F_o \) in (34).

B. Channel Estimation
In practice, \( H_d \) has to be estimated using training symbols and then an estimated version of \( H_d \) is plugged into (34) to determine \( F_o \). The training signals can be designed in a variety of ways. The simplest approach is to design the transmitted signals so that only one entry of \( x \) (see (24)) is non-zero in each differential training symbol; the corresponding column of \( H_d \) can then be estimated from the corresponding received differential measurements \( z \) (see (23) [16]). We present numerical results for \( F_o \) based on perfectly known \( H_d \) as well as estimated \( H_d \).

Note from (39) that we also need estimates of \( HH^H \) and \( H_r H_r^H \) to estimate \( \Sigma_w \) for \( F_o \) in (34). For the special case of interest \( H_r = H \) we have
\[ \text{vec}(HH^H) = (H^r \otimes H^H) \text{vec}(I) = H_d \text{vec}(I) \]  \hspace{1cm} (51)
and thus the two matrices can be extracted from \( H_d \).

V. QUASI-COHERENT INTERFERENCE SUPPRESSION
In this section, we show that a quasi-coherent estimate of \( H \) can be obtained from \( H_d \) which can then be used for linear interference suppression on direct measurements \( r \) and \( r_r \) (rather than on \( z = \text{vec}(rr_r^H) \)) followed by differential detection from appropriate elements of \( z \).

A. Linear Interference Suppression at the Receiver
We have the following channel decomposition of \( H \)
\[ H = H_o \Lambda_\phi \]  \hspace{1cm} (52)
where \( H \) is the actual channel matrix
\[ H = \begin{bmatrix} |h_{11}|e^{j\theta_{h_{11}}} & |h_{12}|e^{j\theta_{h_{12}}} \\ |h_{21}|e^{j\theta_{h_{21}}} & |h_{22}|e^{j\theta_{h_{22}}} \end{bmatrix} \]  \hspace{1cm} (53)
and \( H_o \) is what we can estimate from \( H_d \)
\[ H_o = \begin{bmatrix} |h_{11}| & |h_{12}|e^{j(\theta_{h_{12}} - \theta_{h_{22}})} \\ |h_{21}|e^{j(\theta_{h_{21}} - \theta_{h_{11}})} & |h_{22}| \end{bmatrix} \]  \hspace{1cm} (54)
and \( \Lambda_\phi \) is a diagonal matrix (that is unknown)
\[ \Lambda_\phi = \text{diag}(e^{j\theta_{h_{11}}}, e^{j\theta_{h_{22}}}) . \]  \hspace{1cm} (55)
To see how \( H_o \) can be estimated from \( H_d \), refer to (27). The first column of \( h_{11}^d H_o h_{11}^d \) yields the first column of \( H_o \). Similarly, the second column of \( h_{22}^d H_o h_{22}^d \) yields the second column of \( H_o \).

The MMSE filter matrix in this case is given by
\[ F = H^H (\rho H H^H + \sigma^2 I_n)^{-1} \]  \hspace{1cm} (56)
\[ = \Lambda_\phi^H H_o (\rho H_o H_o^H + \sigma^2 I_n)^{-1} = \Lambda_\phi^H F_o \]  \hspace{1cm} (57)
which operates on the baseband signal vector \( r \). We note that \( F_o \) in (56) is what can be computed at the receiver and used for interference suppression. Thus, processed signal vector from which the differentially encoded symbols are detected is given by
\[ y = F_o r = F_o H s + F_o v . \]  \hspace{1cm} (58)
We note the use of \( F_o \) (rather than \( F \)) does not impact the ability to detect differential symbols since the \( i \)-th differentially encoded transmitted symbol in \( s_i, s^*_{i-r} \) is detected from the product \( y_i y^*_r \). This corresponds to detecting the differentially encoded symbol vector via \( y \circ y^*_r \) where \( \circ \) denotes the Hadamard (element-wise) product.

B. Linear Interference Suppression at the Transmitter
Interference suppression using precoding at the transmitter is another attractive possibility. It turns out that \( H_o \) estimated at the receiver and fed back to the transmitter cannot be exploited due to the phase ambiguity. However, in reciprocal channels, if the transmitter first acts a receiver and estimates the channel matrix from differential measurements (based on training symbols from the receiver), it turns out that it results in the following decomposition of \( H \)
\[ H = \Lambda_\phi H_o \]  \hspace{1cm} (59)
In this case the transmitted signal is precoded as \( s \rightarrow G s \) where [4], [17]
\[ G = \alpha F , \; \alpha = \sqrt{\rho / \text{tr}(F \Lambda_\phi F^H)} \]  \hspace{1cm} (59)
\[ F = (H^H H + \zeta I)^{-1} H^H , \; \zeta = \sigma^2 / \rho , \]  \hspace{1cm} (59)
and \( s \) is the transmitted symbol vector, \( \rho \) represents transmit power (SNR if \( \sigma^2 = 1 \)) per data stream, and \( \Lambda_\phi = E[s s^H] \) is the diagonal covariance of transmitted symbols, which in our case is \( \Lambda_\phi = I \). The composite system matrix with precoding is given by
\[ r = H G s + v \]  \hspace{1cm} (60)
and the composite matrix \( H G \) controls the interference. Note that in terms of \( H_o, F \) is given by
\[ F = (H_o^H H_o + \zeta I)^{-1} H_o^H \Lambda_\phi^* = F_o \Lambda_\phi^* \]  \hspace{1cm} (61)
differential signaling, and the receiver can directly detect the symbols differentially from \( z = \text{vec}(rr_H^r) \) since interference suppression is done at the transmitter.

VI. Numerical Results

In this section, we present numerical results to illustrate the performance of the proposed D-MIMO transceiver architectures for an \( n \times n \) MIMO system with \( n = 2 \) antennas.

Fig. 1 shows a diagram of the D-MIMO MMSE receiver (34), discussed in Sec. IV, that operates on the \( 4 \times 1 \) differential measurements \( z = \text{vec}(rr_H^r) = r^*_e \otimes r_e \) to detect the differential symbols. Results based on uncoded QPSK differential transmission for this receiver are presented in Fig. 2. The figures plot the probability of error \( P_e \) versus SNR for two D-MIMO MMSE receivers: one based on perfect channel state information (CSI) - perfect knowledge of \( H_d \), and one based on estimated \( H_d \) where the estimation is done via training symbols at the same SNR as that for data communication. The performance of a third D-MIMO receiver without interference suppression (\( F_o = I_{n,2} \)) is also shown for comparison. Finally, the performance of two ideal systems is shown for baseline comparison in which there is no interference: \( H \) is diagonal - see Remark 3. One is a coherent system corresponding to two non-interfering QPSK data streams, and the other is a corresponding differential system. The coherent system has the best performance and differential system has a 3dB loss compared to coherent system. The D-MIMO system with perfect CSI is next in line, followed by the D-MIMO with estimated channel. The worst performance is that of D-MIMO without interference suppression. Fig. 2(a)-(c) show the performance of the five systems for 3 different levels of interference. In Fig. 2(a), the interference is strongest: \( |h_{12}|^2 \) and \( |h_{21}|^2 \) are 3dB below \( |h_{11}|^2 = |h_{22}|^2 \), whereas in (b) the interference is 6dB below signal, and in (c) 10dB below signal. The \( P_e \) is computed numerically from 1000,000 symbols, and the phases of the entries of \( H \) change randomly every 1000 symbols. As evident, the D-MIMO receivers can deliver very competitive performance, whereas ignoring interference (D-MIMO w/o interference suppression) can result in unacceptably high \( P_e \).

![Fig. 1: D-MIMO MMSE receiver diagram](image)

In (a) \( r \rightarrow z \rightarrow F_o \rightarrow B \rightarrow A \otimes B \rightarrow z^* \otimes r \rightarrow \Delta \phi \)

\( A \circ B \) denotes the Hadamard product. Results parallel to Fig. 2 for this receiver are shown in Fig. 4. The two baseline ideal receivers, coherent and differential without interference, are the same and so is the D-MIMO receiver without interference suppression. The only difference is in the D-MIMO receivers with interference suppression: in this case, they are quasi-coherent linear MMSE receivers with perfect CSI (\( H \)) and with estimated \( H \). The general trend is the same as in Fig. 2 and the performance of the two D-MIMO MMSE receivers is very comparable. The main difference seems to be that the D-MIMO receivers based on estimated \( H_d \) perform slightly worse than those based on estimated \( H_o \).

![Fig. 2: \( P_e \) versus SNR for various receivers for different levels of interference power using interference suppression on differential measurements: (a) 3dB below signal, (b) 6dB below signal, (c) 10dB below signal.](image)

VII. Conclusion

We have presented two promising D-MIMO transceiver architectures that enable interference suppression in conjunction with spatially multiplexed differential signaling. While
we have not explicitly discussed it, the proposed transceivers can deal with small frequency offsets as well. The results presented in this paper are based on a sub-system (19) of the general D-MIMO model in (15) and (16) that offers a rich structure and array of possibilities for further research. Extensions to multiuser transceivers and wideband scenarios that explicitly account for multipath propagation is another fruitful direction. The sampled approach to wideband MIMO channel modeling in [18] could be particularly relevant in this context. Development of the D-MIMO concept in beamspace for high-dimensional MIMO systems [2]–[4], such as those encountered at mmW frequencies is also a promising direction. Finally, we note that at high frequencies such as mmW, the differential measurements at the receiver can also be realized in an analog fashion using interferometers thereby obviating the need for a local oscillator at the receiver [19].

REFERENCES


Fig. 4: $P_e$ versus SNR for various receivers for different levels of interference power and using quasi-coherent interference suppression: (a) 3dB below signal, (b) 6dB below signal, (c) 10dB below signal.

Fig. 3: Quasi-coherent linear MMSE receiver diagram

\[ r - F_o \begin{bmatrix} y \\ z^{-1}(\cdot) \\ A \otimes B \end{bmatrix} \]

\[ \Delta \phi \]

\[ \text{Symbol Detector} \]